Goodwin cycles and the BoPC growth paradigm: A macrodynamic model of growth and fluctuations

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Abstract

The paper builds a non-linear macrodynamic model to study the relation between the functional distribution of income, technological progress and economic growth. In the short-term, the interaction between the productivity regime, the demand regime and the distributive conflict generates cyclical paths *a la* Goodwin. In the long-term, output growth rate is constrained by the balance of payments *a la* Thirlwall, in which the elasticities of foreign trade are modeled as a function of the complex relation between the wage-share and the innovation capabilities of the economy.

Keywords: Cyclical growth, Goodwin cycles, Distributive cycles, Thirlwall's law, Balance of payments constraint.

JEL: E12, E32, O11.

Área temática: Economia.

1. Introduction

A fundamental characteristic of the capitalist economy is its cyclical and irregular growth behavior. The heterodox tradition in macroeconomics has a variety of models which seek to capture this cycle-tendency relation, giving different emphasis to specific characteristics of the real world. An important exercise has to be done in order to solve the fundamental differences between then, delimiting a basic structure for the dynamics of accumulation and distribution.

The relation between effective demand and income distribution is a central aspect in the heterodox theories of distributive conflict (Barbosa-Filho and Taylor, 2006). The effective demand influences the functional distribution of income through fluctuations in nominal wages and labor productivity. Income distribution in turn influences consumption and investment through cyclical changes in the level of capacity utilization and the wage-share. Distributive conflict models explain inflation, distribution and growth juxtaposing wage demands and price behavior where each part seeks to protect its share on income (Rezai, 2012).

In the other hand, Thirlwall's law, one of the most successful empirical regularities in non-conventional growth theory, proposes that in the long run the main constraint to growth is in the balance of payments (BP) (Thirlwall and Hussain, 1982; Alonso and Garcimartín, 1999; Thirlwall, 2011). Since countries cannot finance systematically permanent BP imbalances, there is an adjustment in aggregate demand that constrain its expansion and consequently output's growth (Setterfield, 2011a; McCombie, 2011).

Taking as inspiration Goodwin's synthesis[1], this study offers a modeling structure that adds up to other efforts in the sense of integrating the key elements of heterodox tradition. The model is compatible with concepts like distributive conflict, autonomous investment function, cumulative causation, balance-of-payments-constraint growth, and the difference between science and technology.

Integrating different economic approaches is always challenging, not just because these contributions are disperse spatially and temporally, but also because they have unique richness and complexity. Efforts in this direction inevitable lead to losses of information as a collateral effect. However, we consider that the exercise is of great benefit since it allows, beyond its mathematical beauty and elegance, a broader view of our study object. It also has pedagogic purposes showing how different concepts can dialogue one with the other.

The paper also explores the interaction between the functional distribution of income, technological progress and economic growth. We built a *KG* (Kaldor-Goodwin) model of endogenous growth that generates cyclical trajectories *a la* Goodwin and a balance-of-payments-constrained growth *a la* Thirlwall. The income elasticities of foreign trade are modeled as a function of the complex relation between the *wage-share* and the innovation capacity of the economy.

The paper's next section is dedicated to present a dynamic non-linear *KG* model with the properties discussed earlier. The last section brings our conclusions and some considerations about future research.

[1] According to Punzo (2006) we can credit to Richard M. Goodwin the great and visionary synthesis in which income distribution, as seen in Marxist analysis, interacts with innovation, as seen by Schumpeter, and the Keynesian effective demand principle, generating typically dynamics of a capitalist economy. The economy is modeled in a way that national production follows the aggregate demand restriction, but the engine of the trajectory is the accumulation made possible by innovation (DiMatteo e Sordi, 2015).

2. A macrodynamic model of growth and fluctuations

There are two ways to formalize growth and fluctuations. The first one consists in build two different theories in which short and long run are independent. The other takes cycle and tendency as indissolubly fused being generated from a unique dynamic system. Our approach is in the middle of these alternatives. In one hand we present separate theories to explain cycle and long-run growth. However, even though not fully integrated, short and long run interact through an adjustment mechanism capable to also generate permanent fluctuations.

The exercise developed in this section explicitly incorporates the principle of effective demand and the existence of distributive conflict in a non-linear macrodynamic model. Since we are working with a "real" economy in the sense that there is no money, the existence of fundamental uncertainty is treated implicitly in the determination of social conventions that sustain the current institutional framework.

Our model uses one of the conceptual ingredients of the classical macrodynamics, namely, the adoption of a formalized structure formed by a collection of functional relations with given parameters[2]. However, inspired in Harrod and Goodwin, we reject the idea of an inherent stable economy in which the dynamic is generate purely by an exogenous impulse that activates a propagation mechanism, that is, the structure of the system.

The model is structurally unstable in two dimensions[3]. First, because of its non-linearity, it generates the possibility of bifurcations. Second because it is subject to continuous disturbances that come from the interaction between the short and long run dynamics.

Following Setterfield and Cornwall (2002) the model is based in three pillars: (i) Productivity Regime; (ii) Demand Regime and (iii) Distributive conflict. We will proceed by presenting each pillar, and then we will present the set of equations that form the dynamic system.

2.1 The Productivity Regime

Let us consider an economy with the follow aggregate production function[4]:

$$X_t = F(K_t; L_t) = \min\left\{\frac{K_t}{a_t} u_t; \frac{L_t}{b_t}\right\} \tag{1}$$

Where X_t corresponds to total output and results from the combination of capital, K_t, and labor, L_t, weighted by their technical coefficients, a_t and b_t. Variable u_t stands for the level of capacity utilization and it is equal to the ratio of current *Y*, and potential output, *Y**. If inputs are efficiently used then the economy must be operating with a level of output that satisfies the following condition:

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[2] For a review about the classical research program in macrodynamics see Punzo (2009).

[3] For a discussion about the structural instability in macrodynamic models see Vercelli (1985; 2000) and Sordi and Vercelli (2006).

[4] Even though heterodox authors usually reject the neoclassical production function and even avoid the utilization of the production function concept itself, we can argue that implicitly is adopted a Leontief type. This proposition comes from the assumption that output's growth rate equals capital's accumulation growth rate or the sum between labor productivity and population growth rates.

$$X_t = \frac{K_t}{a_t} u_t = \frac{L_t}{b_t}$$

In this exercise the production function has two main purposes. First of all it determines the balance condition between capital accumulation and the labor productivity growth. Second, the productivity regime based on the Kaldor-Verdoorn (KV) law depends on it.

In steady state, the level of capacity utilization cannot growth or diminishes indefinitely. This does not mean that u_t is constant. As we will show in the next pages it has its own dynamics and may present permanent fluctuations around a certain equilibrium level. However, in steady state we can approximate $\frac{\partial u}{\partial t}$ to zero. Also taking the technical coefficient of capital as constant, in terms of rates we have

$$\frac{\dot{X}}{X_t} = \frac{\dot{K}}{K_t} = \frac{\dot{L}}{L_t} + \frac{\dot{q}}{q_t} = y \qquad (2)$$

Where q_t corresponds to labor productivity and is given by the inverse of labor technical coefficient ($q_t = 1/b_t$). The supply side equilibrium condition establishes that $\frac{\dot{K}}{K_t} = \frac{\dot{L}}{L_t} + \frac{\dot{q}}{q_t}$.

In order to capture the relation between economic growth and increasing returns of scale we state a linear formulation of the Kaldor-Verdoorn's law:

$$\frac{\dot{q}}{q_t} = \Omega_0(T_t) + \Omega_1 y \qquad (3)$$

Where Ω_0 represents productivity gains *disembodied*, Ω_1 correspond to Verdoorn's coefficient[5] and T_t is a variable that captures the technological conditions of the economy.

Substituting (3) in (2) we obtain the employment growth rate as a difference of the accumulation labor productivity growth rates. Thus:

$$\frac{\dot{L}}{L_t} = (1 - \Omega_1)y - \Omega_0 \qquad (4)$$

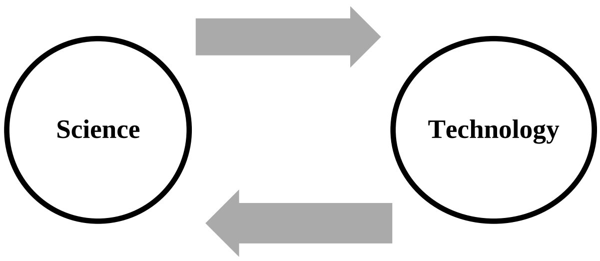
Employment adjusts to the difference between the output's growth rate, given by capital accumulation, and the productivity growth rate. The last one depends on the accumulation itself through increasing returns of scale and also on *disembodied* technological change.

Before continue we have to make some considerations about how technology works in this economy. According to Bernardes and Albuquerque (2003) the National Innovation Systems literature emphasizes the existence of an institutional division of labor between science and technology[6]. In general lines, while universities and research institutes

[5] Recently McCombie and Spreafico (2015) have argued that the intercept cannot and should not be interpreted as an exogenous technical change contribution to growth, and Verdoorn's coefficient does not represent increasing returns *per se*. However, we will follow the traditional interpretation given to both components.

[6] The complex network of interactions and cooperation between agents that contribute to innovation – researchers, engineers, suppliers, producers, users and institutions – while the technological system evolves in a National State has been conceptualize as National System of Innovation (NSI) (Lundvall,

produce science, firms produce technology[7]. Both groups interact and influence each other:



Even though we recognize that the relation between science and technology is not linear, we propose a simple system in order to represent the complex interaction that we understand the Schumpeterian literature suggests exists between those variables. So, be:

$$TECH = \text{ꚍ}_0 + \text{ꚍ}_1 SCIE \qquad (5)$$

$$SCIE = \text{л}_0 + \text{л}_1 TECH \qquad (6)$$

Where $TECH$ represents the technological production and $SCIE$ the scientific infrastructure. The parameters ꚍ_1 and л_1 capture the sensibility of $TECH$ to changes in $SCIE$ and the sensibility of $SCIE$ to variations in $TECH$, respectively. Finally ꚍ_0 and л_0 are exogenous effects.

Solving the system formed by equations (5) and (6) we have that:

$$TECH^* = \frac{\text{ꚍ}_0 + \text{ꚍ}_1 \text{л}_0}{1 - \text{ꚍ}_1 \text{л}_1} \qquad (7)$$

$$SCIE^* = \frac{\text{л}_0 + \text{л}_1 \text{ꚍ}_0}{1 - \text{ꚍ}_1 \text{л}_1} \qquad (8)$$

From the combination of $TECH^*$ and $SCIE^*$ we obtain T_t that represents the technology conditions (or capabilities) of the economy, so $T_t = T(TECH^*; SCIE^*)$. The degree of technological development gathers the vector of capabilities of an economy and determines the trajectories that firms can chose in the migration process to more complex productive structures (Hidalgo et al, 2007; Hidalgo e Hausmann, 2011).

2.2 The aggregate demand curve

The accounting identity of aggregate demand for an open economy without government is given by:

1992; Perez, 2010). Following Metcalfe (1995), the NSI corresponds to the conjunction of institutions that contribute to the development and diffusion of new technologies and operates as a referential that government uses in order to formulate innovation policies.

[7] Technology influences science through several channels that include but are not limited to the formation of a research agenda, as empirical knowledge repository, and source of equipment and research instruments (Rosemberg, 1982). On the other hand, science influences technology as a source of technological opportunities and through labor market (Pavitt, 1991; Klevorick et al, 1995). Ribeiro et al (2010) and Castellacci and Natera (2013) suggest that the channels connecting the scientific infrastructure and the technological production change in coevolution along the growth path.

$$Y_t \equiv C_t + I_t + XL_t$$

Where Y_t corresponds to total output on the demand side, C_t is consumption, I_t is investment, and XL_t corresponds to net exports. Dividing this expression by the capital stock in *t* we have:

$$\frac{Y_t}{K_t} = \frac{C_t}{K_t} + \frac{I_t}{K_t} + \frac{XL_t}{K_t} \tag{9}$$

Our economy has two social classes, namely, workers and entrepreneurs (or capitalists). Workers consume all their income and the entrepreneurs save part of their income. Total consumption is given by:

$$C_t = v_t L_t + c_K r_t K_t \tag{10}$$

Where v_t corresponds to real wages, c_K is the propensity to consume of entrepreneurs, and r_t corresponds to the profit rate. Total wages are given by $v_t L_t$ while total profits are given by $r_t K_t$. Defining the wage-share as $\varpi_t = \frac{v_t L_t}{Y_t}$ we can rewrite equation (10) as:

$$C_t = \varpi_t Y_t + c_K (1 - \varpi_t) Y_t \tag{11}$$

Dividing by the capital stock we have:

$$\frac{C_t}{K_t} = [c_K + (1 - c_K)\varpi_t] u_t \tag{12}$$

Where $u_t = \frac{Y_t}{K_t}$ and the capital technical coefficient was normalized to 1.

Investment in the Kaleckian tradition is represented as a linear function of the profit-share and the level of capacity utilization (Bhaduri e Marglin, 1990). But since the profit-share is the complementary of the wage-share we write:

$$\frac{I_t}{K_t} = \gamma_t - \gamma_1 \varpi_{t-1} + \gamma_2 u_{t-1} \tag{13}$$

Where γ_t represents the autonomous investment component, γ_1 captures the investment sensibility to wage-share variations, and γ_2 captures the sensibility of investment to variations in the level of capacity utilization. An increase in the wage-share reduces investment while an increase in the level of capacity utilization always increases it. As we will show latter γ has a subscript *t* since autonomous investment has its own dynamic.

Equation (13) has two fundamental differences in relation to the usually employed in the Kaleckian growth literature. Investment is not a function of the profit-share but instead responds to the wage-share. This allows us to standardize the model as usually done by the literature of cycles that follows Goodwin (e.g. Goodwin, 1967; Keen, 1995; Barbosa-Filho and Taylor, 2006; Rezai, 2012; Sordi and Vercelli, 2014).

A second difference concerns the temporal position of the variables. While consumption for example depends on the current functional distribution of income and the current level of capacity utilization, we considerer that investment depends on ϖ and u in $t-1$. The economic intuition for that is in the nature of investment. Since I is a crucial variable that links aggregate demand/supply and short/long run we consider that entrepreneurs planned the investment with a lag of one period. So, investment in t results of a decision taken in $t-1$.

Net exports are modeled following Oreiro and Araújo (2013) and the properties describe by Bhaduri and Marglin (1990) and Porcile and Lima (2013). In this way we have:

$$\frac{XL_t}{K_t} = \xi_0 + \xi_1\varepsilon_t - \xi_2 u_t + \xi_3 u^f_{t-1} \tag{14}$$

Where ξ_0 is a constant, ε_t corresponds to real exchange rate and is exogenous, and ξ_1, ξ_2 and ξ_3 are sensibility parameters. Currency devaluation allows an increase in exports and a reduction of imports increasing net exports[8]. An increase in the domestic capacity utilization level increases imports reducing net exports. Finally an increase in foreign capacity utilization increases net exports.

Equation (14) presents an important difference in relation to the formulation usually employ in literature. Net exports in *t* depend on the foreign capacity utilization level in $t-1$. The intuition for this formulation is that while the decision to import is immediate, the decision to export demands planning. The entrepreneurs look to the foreign level of capacity utilization in one period to decide if export the next one.

Substituting equations (12), (13) and (14) in (9) we obtain the aggregate demand as a proportion of the capital stock:

$$u_t = [c_K + (1-c_K)\varpi_t]u_t + \gamma_t - \gamma_1\varpi_{t-1} + \gamma_2 u_{t-1} + \xi_0 + \xi_1\varepsilon_t - \xi_2 u_t + \xi_3 u^f_{t-1} \tag{15}$$

Rearranging the expression and isolating u_t we find the level of capacity utilization as a function of the wage-share and the capacity utilization of the last period:

$$u_t = \frac{\gamma_t + \xi_0 + \xi_1\varepsilon_t - \gamma_1\varpi_{t-1} + \gamma_2 u_{t-1} + \xi_3 u^f_{t-1}}{1 - c_K - (1-c_K)\varpi_t + \xi_2} \tag{16}$$

Advancing equation (16) in one period and subtracting u_t from both sides:

$$u_{t+1} - u_t = \frac{\gamma_t + \xi_0 + \xi_1\varepsilon_t - \gamma_1\varpi_t + \gamma_2 u_t + \xi_3 u^f_t}{1 - c_K - (1-\varpi_t) + \xi_2} - u_t \tag{17}$$

For convenience, the difference equation above can be approximated by a differential equation so we have $\dot{u} = u_{t+1} - u_t$. Calling $\Lambda_u = 1 - c_K - (1-\varpi_t) + \xi_2$, as the inverse of the Keynesian multiplier, then:

$$\dot{u} = \alpha_0 + \alpha_1 u_t + \alpha_2\varpi_t + \alpha_3 u^f_t \tag{18}$$

[8] Taken the Marshall-Lerner condition as granted.

Where $\alpha_0 = \frac{\gamma_t + \xi_0 + \xi_1 \varepsilon_t}{\Lambda_u} > 0$, $\alpha_1 = \frac{\gamma_2 - \Lambda_u}{\Lambda_u} < 0$, $\alpha_2 = \frac{-\gamma_1}{\Lambda_u} < 0$, and $\alpha_3 = \frac{\xi_3}{\Lambda_u} > 0$. Since the Keynesian multiplier is necessary positive, $\Lambda_u > 0$. The Keynesian stability condition demands that $\alpha_1 < 0$, so $\gamma_2 - \Lambda_u < 0$. That means that the sensibility of investment to an increase of the level of capacity utilization has to be lower to the multiplier. It is important to notice that the Keynesian multiplier also depends on the functional distribution of income, $\frac{\partial \Lambda_u}{\partial \varpi_t} > 0$. However, for the sake of simplicity we will take it as constant.

Equation (18) represents our *aggregate demand curve*. Variations in the level of capacity utilization are a function of the income distribution and the domestic and foreign level of capacity utilization. Traditionally aggregate demand curve is obtained through the difference between the desire and guarantee growth rates (e.g. Bhaduri, 2008; Sasaki, 2013; Schoder, 2014). Our exercise proposes an alternative way to derive the problem through the aggregate demand fundamental identity.

2.3 The distributive curve

Models with distributive conflict try to explain inflation and distributive aspects juxtaposing the demands for increases in nominal wages and the price behavior in a way that workers and capitalists try to protect their income share (Rezai, 2012). In our model capitalists are responsible for fixing prices and workers for changes in nominal wages. So we have:

$$\frac{\dot{w}}{w_t} = \lambda_0 + \lambda_1 u_{t-1} + \lambda_2 \frac{\dot{q}}{q_t} + \lambda_3 \frac{\dot{p}}{p_t} \tag{19}$$

$$\frac{\dot{p}}{p_t} = \zeta_0 + \zeta_1 u_{t-1} - \zeta_2 \frac{\dot{q}}{q_t} + \zeta_3 \frac{\dot{w}}{w_t} \tag{20}$$

Where $\frac{\dot{w}}{w_t}$ is the rate of change of nominal wages and $\frac{\dot{p}}{p_t}$ represents inflation. The parameters λ_1 and ζ_1 capture the sensibility of wages and inflation to changes in capacity utilization, respectively. Parameters λ_2 and ζ_2 represent the sensibility of wages and inflation to changes in labor productivity. While an increase in productivity increases wages, the effect on prices is the opposite. Distributive conflict is represented by the capacity of workers to replenish inflation, weighted by coefficient λ_3, and the capacity of capitalists to replenish wage increases, weighted by coefficient ζ_3. Finally, λ_0 and ζ_0 are exogenous parameters that capture the other components of distributive conflict.

Traditionally both $\frac{\dot{w}}{w_t}$ and $\frac{\dot{p}}{p_t}$ are modeled as functions of the difference between the current functional distribution of income and the distribution desire by each social class. We do not agree with this specification. In fact, unions and capitalists are not conscious, explicitly or implicitly, of the level of income concentration. However, they do look directly to the level of capacity utilization, to the adjustment in prices and wages, and to productivity gains.

The relation between entrepreneurs and workers in the distributive conflict depends on the capacity of appropriation of the fruits of technical progress, i.e. the increases in labor productivity, for each group. We have already shown through the Kaldor-

Verdoorn mechanism that $\frac{\dot{q}}{q_t} = \Omega_0 + \Omega_1 \frac{\dot{K}}{K}$. But since $\frac{\dot{K}}{K} = \frac{I}{K}$ and employing equation (13) we have that:

$$\frac{\dot{q}}{q_t} = \Omega_0 + \Omega_1(\gamma_t - \gamma_1 \varpi_{t-1} + \gamma_2 u_{t-1}) \tag{21}$$

Substituting equations (20) and (21) in (19)

$$\frac{\dot{w}}{w} = \frac{\Phi_1}{\Lambda_w} + \frac{\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\Omega_1\gamma_2}{\Lambda_w} u_{t-1} + \frac{(\lambda_3\zeta_2 - \lambda_2)\Omega_1\gamma_1}{\Lambda_w} \varpi_{t-1} \tag{22}$$

where $\Lambda_w = 1 - \lambda_3\zeta_3 > 0$ corresponds to the inverse of the wages multiplier and $\Phi_1 = \lambda_0 + (\lambda_2 - \lambda_3\zeta_2)(\Omega_1\gamma_t + \Omega_0) + \lambda_3\zeta_0$ is a constant term. Equation (22) gives the nominal wages growth rate as a function of the level of capacity utilization and the wage-share.

Substituting equation (21) and (22) in (20) we obtain the inflation rate:

$$\frac{\dot{p}}{p} = \Phi_2 + \left\{\zeta_1 - \zeta_2\Omega_1\gamma_2 + \frac{\zeta_3[\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\Omega_1\gamma_2]}{\Lambda_w}\right\} u_{t-1} + \left[\frac{\zeta_3\Omega_1\gamma_1(\lambda_3\zeta_2 - \lambda\ \ )}{\Lambda_w} + \zeta_2\Omega_1\gamma_1\right] \varpi_{t-1} \tag{23}$$

where $\Phi_2 = \zeta_0 - \zeta_2(\Omega_1\gamma_t + \Omega_0) + \zeta_3 \frac{\Phi_1}{\Lambda_w}$ is a constant term. Equation (23) gives the inflation rate as a function of the level of capacity utilization and the wage-share.

We have defined the wage-share as $\varpi_t = \frac{v_t L_t}{Y_t}$. But we know that real wages are defined by $v_t = \frac{w_t}{p_t}$, that is, the ratio between nominal wages and the price level. On the other hand, $\frac{L_t}{Y_t} = \frac{1}{q_t}$. So in terms of change rates we can write:

$$\frac{\dot{\varpi}}{\varpi_t} = \frac{\dot{w}}{w_t} - \frac{\dot{p}}{p_t} - \frac{\dot{q}}{q_t} \tag{24}$$

Substituting equations (21), (22) and (23) in (24) we have that:

$$\frac{\dot{\varpi}}{\varpi_t} = \frac{\Phi_1}{\Lambda_w} - \Phi_2 - (\Omega_1\gamma_t + \Omega_0) + \left\{\frac{(1-\zeta_3)[\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\Omega_1\gamma_2]}{\Lambda_w} - (1 - \zeta_2)\Omega_1\gamma_2 - \zeta_1\right\} u_{t-1} + \frac{\Omega_1\gamma_1}{\Lambda_w}[(1 - \zeta_3)(\lambda_3\zeta_2 - \lambda_2) + (1 - \zeta_2)\Lambda_w]\varpi_{t-1} \tag{25}$$

Considering small time intervals equation (25) can be rewrite for convenience in continuous form as:

$$\dot{\varpi} = \varpi(\beta_0 + \beta_1 u_t + \beta_2 \varpi_t) \tag{26}$$

Where $\beta_0 = \frac{\Phi_1}{\Lambda_w} - \Phi_2 - (\Omega_1\gamma_t + \Omega_0) \lesseqgtr 0$, $\beta_1 = \frac{(1-\zeta_3)[\lambda_1 + \lambda_3\zeta_1 + (\lambda_2 - \lambda_3\zeta_2)\Omega_1\gamma_2]}{\Lambda_w} - (1 - \zeta_2)\Omega_1\gamma_2 - \zeta_1 \lesseqgtr 0$ corresponds to the sensibility of $\dot{\varpi}$ to changes in the level of capacity utilization, and $\beta_2 = \frac{\Omega_1 \gamma_1}{\Lambda_w}[(1-\zeta_3)(\lambda_3\zeta_2 - \lambda_2) + (1-\zeta_2)\Lambda_w] \lessgtr 0$ is given by the sensibility of $\dot{\varpi}$ to changes in the distribution of income.

Equation (26) represents our *distributive curve*. Variations in the wage-share are modeled as a function of the functional distribution of income and the level of capacity utilization. It is the result of the distributive conflict between capitalists and workers intermediate by the productivity regime of the economy.

2.4 The Distributive System

Suppose the existence of two regions or countries[9]. The global distributive system is formed by the distributive and demand curves of each region:

$$\dot{u} = \alpha_0 + \alpha_1 u_t + \alpha_2 \varpi_t + \alpha_3 u_t^f$$
$$\dot{u^f} = \alpha_0^f + \alpha_1^f u_t^f + \alpha_2^f \varpi_t^f + \alpha_3^f u_t$$
$$\frac{\dot{\varpi}}{\varpi} = \beta_0 + \beta_1 u_t + \beta_2 \varpi_t$$
$$\frac{\dot{\varpi^f}}{\varpi^f} = \beta_0^f + \beta_1^f u_t^f + \beta_2^f \varpi_t^f$$

Where the superscript *f* corresponds to the region consider "foreign". The level of capacity utilization and the wage-share of each region are determined jointly. We assumed that the variables that correspond to the foreign economy are exogenous and included then in the constant term. So, the distributive system can be represented by:

$$\begin{pmatrix} \dot{u} \\ \dot{\varpi}/\varpi \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix} \begin{pmatrix} u_t \\ \varpi_t \end{pmatrix} + \begin{pmatrix} \alpha_0 \\ \beta_0 \end{pmatrix} \tag{27}$$

It is a system of differential equations *2x2* with a linear and a non-linear equation. In *steady-state* $\dot{\varpi} = \dot{u} = 0$. The solution with economic meaning is given by:

$$u^* = \frac{\beta_2}{\beta_1}\left(\frac{\alpha_0\beta_1 - \alpha_1\beta_0}{\alpha_1\beta_2 - \alpha_2\beta_1}\right) - \frac{\beta_0}{\beta_1} \tag{28}$$

$$\varpi^* = \frac{\alpha_0\beta_1 - \alpha_1\beta_0}{\alpha_1\beta_2 - \alpha_2\beta_1} \tag{29}$$

To investigate the stability of the system, we linearized it around the fixed point and named it "implicit equilibrium":

$$\begin{pmatrix} \dot{u} \\ \dot{\varpi} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} u - u^* \\ \varpi - \varpi^* \end{pmatrix} \tag{30}$$

$$J_{11} = \alpha_1 < 0 \tag{31}$$

$$J_{12} = \alpha_2 < 0 \tag{32}$$

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[9] The growth path of an economy describes the process of income creation inserted in a specific historic and institutional context. That means that the economies are structurally distinct between then. There are two ways to represent their differences. The first one rests on the assumption that is possible to model both economies using a unique model with distinct parameters. The second considers that we need a specific model for each economy. In this study we adopted the first strategy.

$$J_{21} = \beta_1 \varpi^* \lesseqgtr 0 \quad (33)$$
$$J_{22} = 2\beta_2 \varpi^* + \beta_1 u^* + \beta_0 \lesseqgtr 0 \quad (34)$$

Assumption 1: The condition to topological equivalence between the systems is satisfied, that is, $\alpha_0\beta_1 \neq \alpha_1\beta_0$.

A system of non-linear differential equations, N, can me mapped by a linear equivalent, L, so that the qualitative properties of L in the neighbor of the critical point are similar of N in the same point. In this case we say that both systems are topological equivalent. It is required that the Jacobian matrix must be invertible (Shone, 2002). A sufficient condition for that is $\det(J) \neq 0$. After some algebreic manipulations we can show that it holds if $\alpha_0\beta_1 \neq \alpha_1\beta_0$.

Proposition 1: The economy is always *profit-led* in its cyclical dynamics.

We say that an economy is *wage-led* if an increase in the wage-share increases the level of capacity utilization. In equilibrium this will happen when $\alpha_2 > 0$. On the other hand, the economy will be *profit-led* if an increase in the wage-share reduces the level of capacity utilization, so that, $\alpha_2 < 0$.

However, since $\alpha_2 = \frac{-\gamma_1}{\Lambda_u} < 0$, the economy is always *profit-led*. This result has important implications in terms of economic policy that we do not discuss here. The distinction between *profit-led* and *wage-led* growth is a major feature of Post-Keynesian economics and it has triggered an extensive econometric literature[10]. From a theoretical perspective our results are in line with the original Goodwin literature and, as we will show in the next section, dialogue with a particular interpretation given by Blecker (2015) to the Kaleckian dilemma. According to Blecker a revision of the empirical studies in the *profit-led vs wage-led* controversies suggests that cyclically the economy is *profit-led* while its tendency is *wage-led*. In any case the intuition is that during the cycle a reduction in the wage-share allows an increase in investment which in turn implies in an increase in the employment and the capacity utilization of the economy.

Proposition 2: If the distributive stability condition holds, that is, $\beta_2 \leq 0$, the system is stable as long as $\alpha_0\beta_1 > \alpha_1\beta_0$.

The stability condition based on Olech's Theorem imposes that $tr(J) < 0$ and $\det(J) > 0$. So we need to have:

(i) $tr(J) \equiv J_{11} + J_{22} = \alpha_1 + \beta_2\varpi^* < 0$
(ii) $\det(J) \equiv J_{11}J_{22} - J_{12}J_{21} = \alpha_0\beta_1 - \alpha_1\beta_0 > 0$.

As a result of the Keynesian and the Distributive stability conditions α_1 and β_2 are negative, so $tr(J) < 0$. In addition, if $\alpha_0\beta_1 > \alpha_1\beta_0$ the second condition will always be satisfied.

Proposition 3: If the distributive stability condition holds whenever $(\alpha_1 + \beta_2\varpi^*)^2 < 4(\alpha_0\beta_1 - \alpha_1\beta_0)$ the critical point will be a node spiral asymptotically stable that in a sense approximates a Goodwin cycle.

[10] For a review of the empirical and theoretical literature in the field see Palley (2014) and Blecker (2015).

In order to analyze this proposition we need to evaluate the nature of the eigenvalues. It depends on the relation between $tr(J)^2$ and $4\det(J)$. We will have a cyclical spiral if $tr(J)^2 < 4\det(J)$ since the eigenvalues will be imaginary. That means that we will have a spiral node as long as $(\alpha_1 + \beta_2\varpi^*)^2 < 4(\alpha_0\beta_1 - \alpha_1\beta_0)$.

Proposition 4: If the distributive stability condition holds and $\beta_0 < 0$, then a distributive adjustment *labor-market-led* is always stable and the *goods-market-led* adjustment is stable as long as $|\alpha_0\beta_1| < |\alpha_1\beta_0|$.

Following Rezai (2012) nomenclature, if an increase in the level of capacity utilization increases $\dot{\varpi}$ we say the economy is *labor-market-led,* so $\beta_1 > 0$. On the other hand the economy will be *goods-market-led* if an increase in the wage-share reduces $\dot{u}$, so that, $\beta_1 < 0$.

As result of the Keynesian stability condition $\alpha_1 > 0$. We also know that $\alpha_0 = \frac{\gamma_t + \xi_0 + \xi_1\varepsilon_t}{\Lambda_u} > 0$ and we are assuming $\beta_0 < 0$. So if $\beta_1 > 0$ then $\alpha_0\beta_1 > \alpha_1\beta_0$ is always true and the system is stable. On the other hand, if $\beta_1 > 0$ then $\alpha_0\beta_1 > \alpha_1\beta_0$ is true as long as $|\alpha_0\beta_1| < |\alpha_1\beta_0|$.

Proposition 5: If the distributive stability holds and $\beta_0 > 0$, then a distributive adjustment *labor-market-led* will be stable as long as $\alpha_0\beta_1 > \alpha_1\beta_0$ and the *goods-market-led* adjustment is always unstable.

As result of the Keynesian stability condition $\alpha_1 > 0$. We also know that $\alpha_0 = \frac{\gamma_t + \xi_0 + \xi_1\varepsilon_t}{\Lambda_u} > 0$ and we are assuming $\beta_0 > 0$. So if $\beta_1 > 0$, as long as $\alpha_0\beta_1 > \alpha_1\beta_0$ the system will be stable. On the other hand, if $\beta_1 < 0$ then $\alpha_0\beta_1 > \alpha_1\beta_0$ is never true and the system will be unstable.

Proposition 6: If the distributive stability condition does not hold and $\alpha_0\beta_1 > \alpha_1\beta_0$ we will have a periodic orbit *a la* Goodwin as long as $\alpha_1 = -\beta_2\varpi^*$.

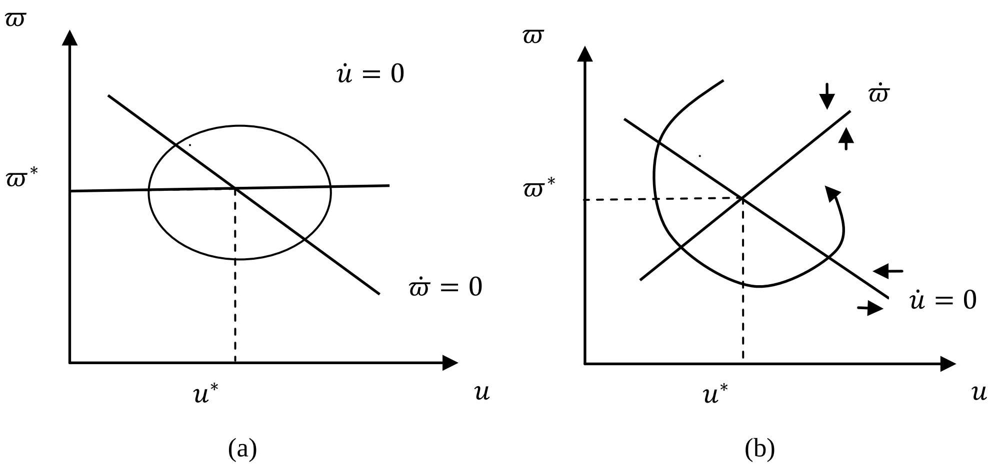
The condition for the appearance of a periodic orbit (or a center) is that $tr(J) = 0$ and $det(J) > 0$. As long as $\alpha_1 = \beta_2\varpi^*$ and $\alpha_0\beta_1 > \alpha_1\beta_0$ both conditions are satisfied.

Proposition 7: If the distributive stability condition does not hold and $\alpha_1 + \beta_2\varpi^* > 0$ the system will always be unstable.

In this case we have $tr(J) > 0$ and Olech's Theorem for stability is violated.

Figure 1 shows the phase portrait of the two cases that generate cyclical motions *a la Goodwin*. Diagram 1a represents the "Periodic Orbit" case while diagram 1b represents the "Spiral node" case in a *labor-market-led* economy.

Figure 1: The Distributive System



Given the values of u^* and ϖ^* determined by equations (28) and (29) we can find the labor productivity, wages, prices, and employment growth rates. For that we substitute (28) and (29) in (21), (22), (3) and (4) respectively. That give us $\frac{\dot{q}}{q}^* (u^*; \varpi^*)$, $\frac{\dot{w}}{w}^* (u^*; \varpi^*)$, $\frac{\dot{p}}{p}^* (u^*; \varpi^*)$ and $\frac{\dot{L}}{L}^* (u^*; \varpi^*)$.

2.5 The Balance of Payments Constraint

According to Thirlwall's law the BoPC growth rate is given by:

$$y^{BP} = \frac{\varphi}{\rho} y_f \tag{35}$$

Where y^{BP} is the balance-of-payments-constraint growth rate, that is, the growth rate allowed by the aggregate demand constraint[11]. Finally y_f is the growth rate of the foreign region.

Looking to advance in the study of the determinants of the foreign trade elasticities ratio we must address the impact of technological capabilities and income distribution on it. The hypothesis that there is a positive relation between technological capabilities and non-price competitiveness $\left(\frac{\varphi}{\rho}\right)$ is quite obvious and strongly supported theoretical and empirically[12].

On the other hand the relation between inequality and the foreign trade elasticities is not that obvious. Structuralist authors like Furtado (1968) and Tavares and Serra (1976) argue that high levels of income inequality in Latin America led to significant differences in consumption patterns between the lower and upper classes. Upper classes demand superfluous and highly technological products that, as result of its small scale, were incapable to induce domestic production. In this sense Bohman and Nilsson (2007)

[11] Long run growth is directly proportional to the product between the foreign income growth and the ratio between the income elasticities of exports and imports. Growth is balance-of-payments-constrained in the sense that there is a limit of supply currency that the economy can count to satisfy its needs to import. The higher the ratio between the foreign trade elasticities the lower would be the BP constraint.

[12] For a review about the recent literature in this matter see Ribeiro, McCombie and Lima (2015).

and Dalgin et al (2008) conclude that, given the non-homothetic preferences, more unequal countries tend to export relatively more necessity goods and import more luxury goods[13]. We should expect then that a better income distribution improves non-price competitiveness.

Controversy comes from the indirect influence that income distribution may have on the foreign trade elasticities through technology. Acemoglu, Robinson and Verdier (2012) for instance focusing on industrialized economies claim that income inequality is required in order to stimulate innovation. Innovation itself can temporarily raise inequality if innovators dispose of quasi rents (Cozzens, 2008).

Still, Weinhold and Nair-Reichert (2009) analyzed a longer sample of 53 developed and developing countries between 1994 and 2000. They conclude that a more equitable income distribution seems to be positive correlated to innovation via its positive effects on the functioning of domestic institutions. Similar results are provided by Hopkin, Lapuente and Moller (2014) that consider that more equitable systems like the Scandinavian economies perform better that US in terms of innovation[14].

In this matter Ribeiro, McCombie and Lima (2015) propose to model the foreign trade elasticities ratio as a positive linear function of the wage-share and the inverse of the technology gap between foreign and domestic economy. We will follow the assertive that "*statistical evidence generally supports the view that inequality impedes growth [...]*" (Ostry, Berg and Tsangarides, 2014). This is not the same as to assume a positive relation between $\frac{\varphi}{\rho}$ and ϖ (even though the relation exists, as we will show in a non-linear fashion). We suggest that non-price competitiveness change while technological conditions evolve given a wage-share.

Thirlwall (1997) and Setterfield (1997) argued that the elasticity of exports growths as the country moves from the production of primary products to manufactures and decreases when the economy get lock in antiquate industrial structures. As a result we should observe an inverted U. On the other hand, McCombie and Roberts (2002) consider that is the ratio between the foreign trade elasticities that present the inverted U relation. While low growth rates generate pressures to an increase in the elasticities ratio, high growth rates would encourage the lock-in of the productive structure.

In order to capture these insights our approach focus on the trajectory of the elasticities as the domestic technological conditions evolves. Some important properties arise from the interaction between T, ϖ and $\left(\frac{\varphi}{\rho}\right)$. Using a logistic function:

$$\left(\frac{\varphi}{\rho}\right)_{T+1} = G\left[\left(\frac{\varphi}{\rho}\right)_T ; \varpi\right] = \varpi \left(\frac{\varphi}{\rho}\right)_T \left[z - \left(\frac{\varphi}{\rho}\right)_T\right] \qquad (36)$$

Where z corresponds to a technological variable that captures knowledge globally available. The difference equation above takes the ratio between the foreign trade elasticities in $T + 1$ as a function of the wage-share and the elasticities ratio itself in T.

[13] Engel's law states that, as income grows, consumers tend to substitute necessity goods by luxury goods, where the latter have income elasticity of demand greater than unity and the first have income elasticity of demand less than unity. Here, non-homothetic preferences basically mean that the proportion of income that consumers spend on luxury and necessity goods varies as income increases (Ribeiro, McCombie and Lima, 2015).

[14] For a review about the recent literature in this matter see Weinhold and Nair-Reichert (2009) and Botta (2015).

Notice that here T represent the technological conditions of the economy. The productive structure, represented by the foreign trade elasticities ratio, follows an inverted U associated with the distributive and technological conditions.

In the long run, $\left(\frac{\varphi}{\rho}\right)_{T+1} = \left(\frac{\varphi}{\rho}\right)_T = \left(\frac{\varphi}{\rho}\right)^*$. We have two possible solutions, namely, (i) $\left(\frac{\varphi}{\rho}\right)^* = 0$ e (ii) $\left(\frac{\varphi}{\rho}\right)^* = \frac{\varpi z-1}{\varpi}$. Applying Taylor's polynomial to equation (36) we have:

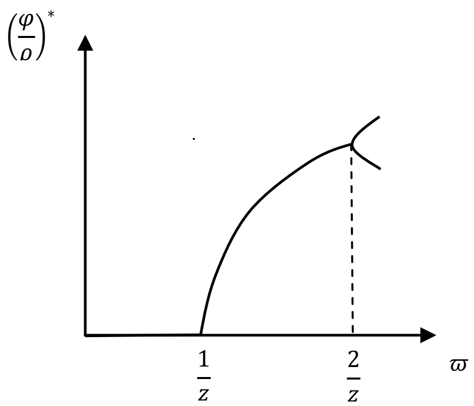
$$\left(\frac{\varphi}{\rho}\right)_{T+1} = G\left[\left(\frac{\varphi}{\rho}\right)^*;\varpi\right] + \frac{\partial G}{\partial\left(\frac{\varphi}{\rho}\right)^*}\left[\left(\frac{\varphi}{\rho}\right)_T - \left(\frac{\varphi}{\rho}\right)^*\right] \tag{37}$$

But we know that $G\left[\left(\frac{\varphi}{\rho}\right)^*;\varpi\right] = \left(\frac{\varphi}{\rho}\right)^*$ and we can show that $\frac{\partial G}{\partial\left(\frac{\varphi}{\rho}\right)^*} = (2-\varpi z)$. Substituting both values in (45) and rearranging the terms:

$$\left[\left(\frac{\varphi}{\rho}\right)_{T+1} - \left(\frac{\varphi}{\rho}\right)^*\right] = (2-\varpi z)\left[\left(\frac{\varphi}{\rho}\right)_T - \left(\frac{\varphi}{\rho}\right)^*\right] \tag{38}$$

Equation (38) can be rewrite as a simple difference equation in the format $l_{T+1} = (2-\varpi z)l_T$. The stability condition demands that $0 < 2-\varpi z < 1$. That implies $\frac{1}{z} < \varpi < \frac{2}{z}$. Figure 2 represents $\left(\frac{\varphi}{\rho}\right)^*$ as a function of ϖ:

Figure 2: Foreign trade elasticities and the functional income distribution



Proposition 8: The points $\varpi = \frac{1}{z}$ and $\varpi = \frac{2}{z}$ are bifurcation points where we observe qualitative changes in the behavior of the objective function.

For $\varpi < \frac{1}{z}$, that is, with a sufficient low wage-share, the productive structure will be in *lock-in*. Technology in this case is not capable to increase the foreign trade elasticities ratio and consequently cannot relieve the external constraint. For $\varpi > \frac{1}{z}$, a better income distribution favoring wages allows a relief of the external constraint while T evolves. However, for $\varpi > \frac{2}{z}$ the function presents chaotic behavior.

There is a security band for ϖ linked to the global technological conditions. It is quite reasonable to assume that a wage-share too high reduces the investment capacity of entrepreneurs compromising long run growth. At the same time a wage-share too low can conduct the economy to a "demand trap".

Proposition 9: The economy is always *wage-led* in its BoPC dynamics.

Outside the *lock in* case we have that $\left(\frac{\varphi}{\rho}\right)^* = \frac{\varpi z - 1}{\varpi}$ and $\frac{\partial\left(\frac{\varphi}{\rho}\right)^*}{\partial \varpi} > 0$. An increase in the *wage-share* allows an increase in the BoPC growth rate. Therefore the economy is in a sense *wage-led* in its BoPC dynamics.

We shall notice that while cyclically the accumulation regime is always *profit-led,* in the long run the economy is always *wage-led* since the ratio between the foreign trade elasticities is a positive function of the *wage-share.* This is in line with Blecker (2015) that shows that empirical evidence focus in the cycle dynamics usually finds *profit-led* results, while works that focus in aggregate demand find *wage-led* results.

2.6 Growth and fluctuations: When the short-term meets the long run

The model presented gives us two growth rates: (i) the capital accumulation growth rate and (ii) the balance-of-payments-constraint growth rate. Whereas both of them are given quite independently, they can be equal just for coincidence[15]. So we have:

$$y = \gamma_t + \gamma_1 \varpi^* + \gamma_2 u^* \tag{39}$$

$$y^{BP} = \frac{\varphi}{\rho} y_f \tag{40}$$

We need them to explicitly specify an adjustment mechanism between y and y^{BP} in order to avoid the over determination problem.

Since there is plentiful empirical evidence supporting that in the long run growth is balance of payments constrained[16] the follow dynamic is proposed. If $y > y^{BP}$, capital accumulation exceeds the balance-of-payments-constraint and capitalists are forced to reduce y in order to guarantee the BP equilibrium. On the other hand, if $y < y^{BP}$ there is space to expand accumulation in order to approximate y to y^{BP}. Since economic agents are immersed in an environment of fundamental uncertainty in both cases their calculations are subjective.

While the first derivative of y in t corresponds to variations in the rhythm of capital accumulation in time, the second derivative corresponds to the intensity of those variations. Put another way, $\ddot{y}$ captures the intensity of the adjustment of y on time. We consider that the intensity of the adjustment is a linear function of the difference between y and y^{BP}, that is:

$$\ddot{y} = j_t (y^{BP} - y) \tag{41}$$

Where $j_t > 0$ is an exogenous variable that captures the subjective perception of the necessity of adjustment. Equation (41) shows that the higher the difference between y^{BP} and y the higher the intensity of the adjustment will be.

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[15] Since the elasticities are a function of the wage-share, is not strictly correct to say that both growth rates are independently. However, we consider here that the elasticities change slowly and depend more on the evolution of technology given a wage-share that of the wage-share itself.

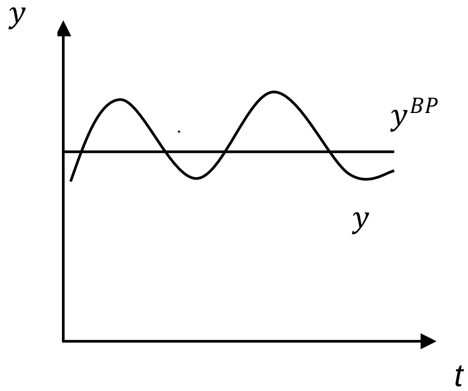
[16] For reviews of the literature, see Thirlwall (2011) and Dávila-Fernández and Amado (2015). In one of the most important and ambitious efforts Gouvea and Lima (2013) tested the law for a sample of 90 countries finding empirical evidence to support Thirlwall's law. Also about the existence of distributive cycles see Sordi and Vercelli (2014) and Arnin and Barrales (2015).

Solving the differential equation we have:

$$y = y^{BP} + sen(t\sqrt{\mathrm{j}}) \tag{42}$$

Figure 3 represents the mechanism so far described:

Figure 3: Adjust mechanism between y^{BP} and y



When $y > y^{BP}$, the capital accumulation growth rate exceeds the external constraint, there has to be an adjustment in investment. In our model it happens through a reduction of γ_t. This could be because the possibility of a current account crisis makes the government or capitalists cuts investments. Since is a decentralized decision and subject to fundamental uncertainty, y will be reduce to the point that $y < y^{BP}$. In some moment and depending on the value of j, capital accumulation starts raising through an increase in γ_t. It is important to notice that changes in γ_t imply in a constant movement of the "implicitly solution". Considering the j_t can change in a non-probabilistic way this dynamic may generates irregular cycles represented in the figure above. This story corresponds basically to a current account crisis mechanism.

Proposition 10: The capitalist system is inherently unstable as result of the interaction between the short-term and long-term dynamics – reflected in y and y^{BP} - that generate continuous fluctuations through autonomous investment, γ_t.

The appropriate way to treat the problem would be include $\gamma_t = \frac{\varphi}{\rho} y_f + sen\left(t\sqrt{j_t}\right) + \gamma_1 \varpi_t - \gamma_2 u_t$ in the distributive system and proceed to the dynamic analysis. Doing that we will fully integrated cycle and tendency and there will be no more distinction between short/long run.

However this leads us to a non-autonomous non-linear dynamic system. We avoid this road even though recognize that further research has to be done in order to enrich and complete the exercise.

Proposition 11: If the conditions establish by Proposition 6 are fulfilled, we will have two endogenous sources of instability. First, the possibility of a periodic orbit between ϖ^* and u^* that generates a cyclical accumulation path. Second, the interaction between y and y^{BP} that also will generate continuous and permanent fluctuations.

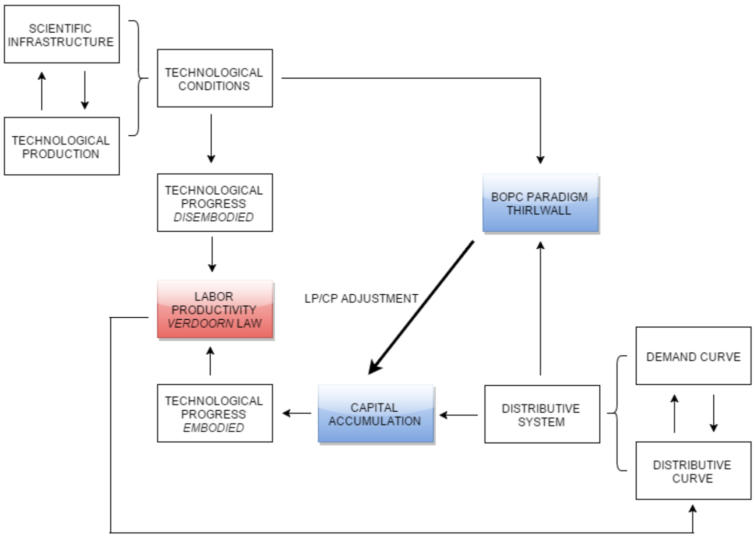
Even though is not possible to eliminate the cyclical fluctuations, the model suggests that a development strategy depends on technological and distributive variables. On the one hand, the strengthening of the scientific and technological capabilities relieves the external constraint on growth and allows a higher rate of capital accumulation. In parallel we recommend a better distribution between capital and labor in order to obtain smooth distributive conflict and avoid distributive growth traps.

There are four key variables that, immersed in a determined institutional context, form the system: (i) Scientific Infrastructure, *SCIEN*; (ii) Technological Production, *TECH*; (iii) Wage-share, ϖ and (iv) Level of capacity utilization, u.

Technological progress expresses itself through increases in labor productivity. However, since part of it cannot be dissociate from the capital accumulation process, we have that capital accumulation itself influences positively labor productivity. Departing from our investment function it depends directly on ϖ and u. Moreover, since growth is balanced-of-payments-constrained it depends indirectly on the *SCIEN* and *TECH*.

From a macroeconomic perspective we suggest that growth and labor productivity are consolidated from two large blocks that interact with each other. The first one appears from the direct link between science and technology (that is, its technological capabilities) that influences the external constraint. The second appears from the indirect link between the functional distribution of income, the level of capacity utilization and the external constraint. The ultimate expression of growth and technological progress is the increase in labor productivity.

Figure 4: A synthetic diagram of the model



3. Conclusion

This paper built a dynamic *KG* model in order to study the relation between functional distribution of income, technological progress and economic growth. In the short run, the distributive conflict between capital and labor interacts with the productivity and demand regimes generating cyclical paths *a la* Goodwin. In the long run, the elasticities of foreign trade were modeled as a function of the technological conditions and the wage-share, so that economic growth is balance-of-payments-constrained.

The model proposes a way in which *lock in* traps could appear associated with high levels of inequality. It allows us to better understand the relation between distributive, technological variables and growth, combining elements of Marxist, Kaleckian, Goodwin and Kaldorian traditions. The main element that unifies this "*strange* *conjunction of stars"* is the conception of the capitalist economy as a structurally unstable system.

The approach proposed here has four main limitations that appear also as opportunities for future research. First, the nature of the model is restricted to a small open economy. It will be interesting to advance in order to have a wider model where growth in the North and the South are determined simultaneously.

Second, technological progress was modeled in a very rudimentary way. The mechanisms through which the scientific infrastructure and the technological production interact, forming the technological capabilities, need to be better explored, following the Schumpeterian tradition. One way to do that could be the establishment of a non-linear interaction between science and technology.

Third, considering that the long and short-run interact permanently, it would be interesting to advance to a model in which the foreign trade elasticities are jointly determined in the distributive system. This development involves some mathematical complications that were avoided here but at the same time would enrich the exercise. The idea is to evolve in order to construct non-autonomous non-linear dynamic systems.

Finally the *KG* model does not consider the monetary/finance face of the economy. Since the Keynesian economy is a Monetary Economy of Production there is no capitalism without money or banks. According to Vercelli (1985) the principal explanation for the structural instability of capitalism is in the properties of money and credit. There have been significant efforts to built *GM* (Goodwin-Minsky) models (e.g. Keen, 1995; Vercelli, 2000; Sordi and Vercelli, 2006; 2012; 2014). A natural next step would be to construct a *KGM* model that incorporates the Minskyan hypothesis of financial fragility.

References

Acemoglu, D., Robinson, J.A., and Verdier, T., 2012. Can we all be more like Scandinavians? Asymmetric growth and institutions in an interdependent world. NBER Working Paper 18441.

Alonso, J.A. and Garcimartín, C., 1999. A new approach to balance-of-payments constraint: some empirical evidence, Journal of Post Keynesian Economics 21(2), 259-281.

Arnin, R. and Barrales, J., 2015. Demand-driven Goodwin cycles with Kaldorian and Kaleckian features. Review of Keynesian Economics 3(3), 351-373.

Barbosa, N. and Taylor, L., 2006. Distributive and demand cycles in the US economy: a structuralist Goodwin model. Metroeconomica 57(3), 389-411.

Bernardes, A.T. and Albuquerque, E.M., 2003. Cross-over, thresholds and interactions between science and technology: lessons for less-developed countries. Research Policy 32(5), 865-885.

Bhaduri, A. 2008. On the dynamics of profit-led and wage-led growth. Cambridge Journal of Economics 32(1), 147-160.

Bhaduri, A. and Marglin, S., 1990. Unemployment and the real wage: the economic basis for contesting political ideologies. Cambridge Journal of Economics, 14(4), 375-393.

Blecker, R., 2015. Wage-led versus profit-led demand regimes: The long and the short of it. Annual conference of the Eastern Economic Association.

Bohman, H. and Nilsson, D., 2007. Income inequality as a determinant of trade flows. International Journal of Applied Economics 4(1), p. 40–59, 2007.

Castellacci, F. and Natera, J.M., 2013. The dynamics of National Innovation Systems: A panel cointegration analysis of the coevolution between innovative capability and absorptive capacity. Research Policy 42(3), 579-594.

Cozzens, S.E., 2008. Innovation and inequality. Georgia Institute of Technology Working Papers Series 30.

Dalgin, M., Trindade, V. and Mitra, D., 2008. Inequality, nonhomothetic preferences, and trade: a gravity approach. Southern Economic Journal 74(3), 747–774.

Dávila-Fernández, M.J. and Amado, A., 2015. Entre a lei de Thirlwall e a hipótese Prebisch-Singer: Uma avaliação da dinâmica dos termos de troca em um modelo com restrição no balanço de pagamentos. Economia e Sociedade 24(1), 87-119.

Di Matteo, M. and Sordi, S., 2015. Goodwin in Siena: Economist, social philosopher and artist. Cambridge Journal of Economics 39(6).

Furtado, C. 1968. Subdesenvolvimento e Estagnação na América Latina. R. Janeiro, Civilização Brasileira.

Goodwin, R.M., 1967. A growth cycle, in: Feinstein, G.H. (Ed.), Socialism, Capitalism and Economic Growth. Cambridge: Cambridge University Press.

Gouvea, R.R, and Lima, G., 2013. Balance of payments constrained growth in a multisectoral framework: a panel data investigation. Journal of Economic Studies 40(2), 240-254.

Hidalgo, C., Klinger, B., Barabasi, A.L. and Hausmann, R., 2007. The product space conditions the development of nations. Science 317(5837), 482-487.

Hidalgo, C. and Hausmann, R., 2011. The network structure of economic output. *Journal of Economic Growth* 16(4), 309-342.

Hopkin, J., Lapuente, V. and Moller, L. 2014. Lower levels of inequality are linked with greater innovation in economies. LSE American Politics and Policy. Access online http://eprints.lse.ac.uk/58473/ 16.09.2015

Keen, S. 1995. Finance and Economic Breakdown: Modeling Minsky's Financial Instability Hypothesis. Journal of Post Keynesian Economics 17(4), 607-635.

Keen, S., 2013. Predicting the 'Global Financial Crisis': Post-keynesian macroeconomics. Economic Record 89, 228-254.

Klevorick, A., Levin, R., Nelson, R. and Winter, S., 1995. On the sources and significance of inter-industry differences in technological opportunities. Research Policy 24, 185-205.

Lundvall, B. 1992. National systems of innovation. New York: Printer Publishers.

McCombie, J., 2002. Increasing returns and the Verdoorn law from a kaldorian perspective, in: McCombie, Pugno and Soro (Eds.), Productivity growth and economic performance: Essays on Verdoorn's law. New York: Palgrave McMillan.

McCombie, J., 2011. Criticism and defences of the balance-of-payments constrained growth model: some old, some new. PSL Quarterly Review 64, 353-392.

McCombie, J. and Roberts, M., 2002. The role of balance of payments in economics growth, in: Setterfield, M. (Ed.), The economics of demanded-led growth: challenging the supply-side vision of the long run. Cheltenham: Edward Elgar.

McCombie, J. and Spreafico, M. 2015. Kaldor's 'technical progress function' and Verdoorn's law revisited, Cambridge Journal of Economics.

Metcalfe, J.S., 1995. Technology Systems and Technology Policy in a Evolutionary Framework, Cambridge Journal of Economics 19(1), 25-46.

Oreiro, J.L. and Araújo, E., 2013. Exchange Rate Misalignment, Capital Accumulation and Income Distribution, Panoeconomicus 3(Special Issue), 381-396.

Ostry, J.D., Berg, A., and Tsangarides, C.G. 2014. Redistribution, inequality, and growth. IMF Staff Discussion Paper 14/02.
Palley, T., 2002. Pitfalls in the theory of growth: an application to the balance-of-payments-constrained growth model, in: Setterfield, M. (Ed.), The economics of demanded-led growth: challenging the supply-side vision of the long run. Cheltenham: Edward Elgar.
Palley, T., 2014. Rethinking wage vs profit-led growth theory with implications for policy analysis. Annual conference of the Research Network in Macroeconomics and Macroeconomic Policies.
Pavitt, K., 1991. What makes basic research economically useful?. Research Policy 20(2), 109-119.
Perez, C., 2010. Technological revolutions and techno-economic paradigms. Cambridge Journal of Economics 34(1), 185-202.
Porcile, G. and Lima, G.T., 2013. Economic growth and income distribution with heterogeneous preferences on the real exchange rate. Journal os Post Keynesian Economics 35, 651-674.
Punzo, L., 2006. Towards a disequilibrium theory of structural dynamics: Goodwin's contribution. Structural Change and Economic Dynamics 17, 382-399.
Punzo, L., 2009. A nonlinear history of Growth and Cycles Theories. History of Political Economy 41, 88-106.
Rada, C. and Talor, L., 2006. Empty sources of growth accounting, and empirical replacements a la Kaldor and Goodwin with some beef. Structural Change and Economic Dynamics 17(4), 486-500.
Rezai, A., 2012. Goodwin cycles, distributional conflict and productivity growth. Metroeconomica 63(1), 29-39.
Ribeiro, L.C., Ruiz, R.M., Bernardes, A.T. and Albuquerque, E.M., 2010. Matrices of science and technology interactions and patterns of structured growth: implications for development. Scientometrics 83, 55-75.
Ribeiro, R.S.M., McCombie, J.S. and Lima, G.T., 2015. Exchange Rate, Income Distribution and Technical Change in a Balance-of-Payments Constrained Growth Model. Working Paper Series, Department of Economics FEA-USP.
Rosemberg, N., 1982. Inside the Black Box: Technology and Economics. Cambridge University Press: Cambridge.
Sasaki, H., 2013. Cyclical growth in a Goodwin-Kalecki-Marx model. Journal of Economics 108, 145-171.
Schoder, C., 2014. Instability, stationary utilization and effective demand: A structuralist model of endogenous cycles. Structural Change and Economic Dynamics 30, 10-29.
Setterfield, M., 1997. History versus equilibrium and the theory of economic growth. Cambridge Journal of Economics 21(3), 365-378.
Setterfield, M., 2011a. The remarkable durability of Thirlwall's Law. PSL Quarterly Review 64(2), 393-427.
Setterfield, M., 2011b. Post-Keynesian macrodynamics and path-dependent growth. European Journal of Economics and Economic Policies: Intervention 8(2), 299-316.
Setterfield, M. and Cornwall, J. A., 2002. A neo-Kaldorian perspective on the rise and decline of the Golden Age, in: Setterfield, M. (Ed.), The economics of demanded-led growth: challenging the supply-side vision of the long run. Cheltenham: Edward Elgar.
Shone, R., 2002. Economic Dynamics, 2nd edition, Cambridge University Press.

Sordi, S. and Vercelli, A., 2006. Financial fragility and economic fluctuations. Journal of Economic Behavior & Organization 61, 543-561.

Sordi, S. and Vercelli, A., 2012. Heterogeneous expectations and strong uncertainty in a minskyian model of financial fluctuations. Journal of Economic Behavior & Organization 83, 544-557.

Sordi, S. and Vercelli, A., 2014. Unemployment, income distribution and debt-financed investment in a growth cycle model. Journal of Economic Dynamics and Control 48, 325-348.

Tavares, M. C. and Serra, J., 1976. Além da estagnação: uma discussão sobre o estilo de desenvolvimento recente do Brasil, in: Serra, J. (Ed.), América Latina, ensaios de interpretação econômica, Rio de Janeiro, Paz e Terra.

Thirlwall, A.P., 1979. The balance of payments constraint as an explanation of international growth rate differences. BNL Quarterly Review 32, 45-53.

Thirlwall, A.P., 1997. Reflections on the concept of balance-of-payments-constrained growth. Journal of Post Keynesian Economics 19(3).

Thirlwall, A.P., 2011. Balance of payments constrained growth models: history and overview. PSL Quarterly Review 64(259), 307-351.

Vercelli, A., 1985. Keynes, Schumpeter, Marx and the structural instability of capitalism. Cathiers d'economic politique 10.

Vercelli, A., 2000. Structural financial instability and cyclical fluctuations. Structural Change and Economic Dynamics 11(1-2), 139-156.

Weinhold, D. and Nair-Reichert, U., 2009. Innovation, inequality and intellectual property rights. World Development 37(5), 889–901.