Applying pure mathematics: IMPA and the dissemination of mathematical economics in Brazil

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Resumo: O objetivo do artigo é contribuir para o entendimento histórico da internacionalização e disseminação da economia matemática no Brasil. Em foco, o papel central do Instituto de Matemática Pura e Aplicada (IMPA) para o desenvolvimento da economia matemática. Avalia-se o porquê da economia tornar-se a disciplina representante da matemática aplicada em um instituto influenciado pelo purismo matemático do grupo Bourbaki. O artigo também explora a prática da teoria econômica dentro de um departamento de matemática, analisando a relação entre estas duas disciplinas.

Palavras-chave: Pensamento Econômico Brasileiro, Economia Matemática, Bourbaki, IMPA, História do Pensamento Econômico

Área: História Econômica, do Pensamento Econômico e Demografia Histórica

O autor agradece à FAPESP pelo financiamento da pesquisa necessária para a realização deste trabalho.

Introduction

Unity and purity are two main features of the mathematics advocated by Nicolas Bourbaki, a fictitious scholar created by a group of mathematicians[1] in 1935. Bourbaki (1950) takes up on the fragmentation of mathematics that followed the increase in the number of works devoted to the discipline. Facing the question of whether this growth would develop a "strongly constructed organism, acquiring ever greater cohesion and unity with its new growths, or whether it is the external manifestation of a tendency towards a progressive splintering" (221) Bourbaki advocates for the existence of a single *mathématique*, based on the axiomatic method and the study of mathematical structures. On the other hand, this single *mathématique* had a defined internal pecking order where pure mathematics – the foundation of the axiomatic method that provided unity to mathematics – held the highest rank. Dieudonné remembered the difference of status of pure and applied mathematics at Bourbakist circles in France: "When one noticed a talented student, one would tell him 'you should do pure math'. On the other hand, one would advise a mediocre student to do applied math while thinking, 'It's all that he can do.'"[2] In a book aiming to present the "pure mathematics as seen by the Bourbaki group", Dieudonné (1982) dedicated a section on each topic to discuss the connections of the pure theory to the natural sciences. Those sections never surpass one paragraph, and in some cases are limited to a succinct phrase: "none at the present". In his autobiography, Laurent Schwartz recognizes that the lack of interest in applied mathematics by the Bourbaki group "caused a historical slowdown in French probability theory", and its influence "prevented applied mathematics from picking up speed in France" (Schwartz 2001, 164).

The dispute between pure and applied mathematics was part of a particular representation dispute on the mathematical field after World War II, what Dahan Dalmedico (2001) calls an 'image war' focused on the cleavage between those two categories. As the war effort raised the demand of applied mathematics, fields as algorithm theory, numerical analysis, partial differential equations, probability and statistics expanded from the pragmatic needs generated by the war effort. In the aftermath of the war, applied mathematics continued its growth, with multiple institutions emerging in the United States dedicated to different fields of applied mathematics.[3] In contrast, the Bourbaki group upheld the primacy of the pure versus the applied.[4] Nonetheless, as Ingrao and Israel (1990), and Weintraub and Mirowski (1994) have argued, Bourbaki mathematics had a defining

[1] The founding members of the Bourbaki Group were Henri Cartan, Claude Chevalley, Jean Coulomb, Jean Delsart, Jean Dieudonné, Charles Ehresmann, Szolem Mandelbrojt, and André Weil. See Mashaal, 2006, 6.

[2] The quotation comes from an interview in Schmidt, 1990; the translation to English is cited in Mashaal 2006, 118.

[3] Dahan Dalmedico (2001, 230) lists The Association for Computing Machineries (1947), The Industrial Mathematical Society (1949), The Operations Research Society of America (1952), The Institute for the Management Sciences (1953) as examples of applied mathematics organizations created in the postwar.

[4] The Bourbakists, however, were not the only mathematicians to advocate the primacy of purity after the war. For an American example of this image of postwar mathematics, see Stone 1957.

influence on the economics done by Gerard Debreu, in particular in *Theory and Value* (1959). As Debreu's mathematical structures were empty of empirical reference, and logically independent of possible interpretations, "the theory may therefore be developed in its formal aspects as a pure mathematical structure" (Ingrao and Israel 1990, 287). Mathematical economics became a singular applied version of Bourbaki mathematics.

The Bourbaki group has a significant influence in the history of mathematics in Brazil during the XX century. Between 1945 and 1952, several founding members of the group have taught in São Paulo and Rio de Janeiro, influencing a young generation of Brazilian mathematicians. In the wake of this event (largely absent in the historical literature on the Bourbaki group), a small department of mathematics was created in Rio de Janeiro in order to provide institutional support for this new internationalized generation of mathematicians. It was named IMPA, portuguese initials to Institute of Pure and Applied Mathematics. However, as a Bourbakist inspired institute, the "applied mathematics" was only present on name at its outset, as members of the institute were interested in topics such as algebraic topology, vectorial topological spaces and number theory. Albeit the efforts of its director, amidst economic and political crisis that limited the institute's budget, the focus of IMPA members on theoretical problems, and even the lack of demand of applied mathematics in a country with limited industrial activity, it took two decades for the creation of a stable group of applied mathematics.

With the expansion of resources of the institute after a period of high growth of the Brazilian economy in the late sixties, IMPA moved to a larger building with enough facilities to expand its activities and then capable of attracting young students interested in mathematical courses. Aloisio Araujo and José Alexandre Scheinkman were part of this community of young students attending courses at IMPA – both would pursue a career in economics after graduate studies in the United States. In the late seventies, mathematical economics was being done in the operations research group at IMPA due the influence of Jack Schechtman, and then in 1980 a graduate program in mathematical economics was created around Araujo, who came back to Brazil to organize the program. The mathematical economics program still exists at IMPA today, and it is still organized around Araujo. Despite its very limited size, the program was able to have an enduring impact on the dissemination of mathematical economics in Brazil and to some extent to the rest of Latin America. Cooperation with other economics departments in Rio, the rigorous mathematical core of disciplines, and the integration of IMPA in international circles were able to create a successful program. Although still a mathematics department, and primarily dedicated to pure mathematics, IMPA turned into an institutional stronghold of mathematical economics in Brazil.

The development of economics at IMPA occurred in a period of internationalization of the discipline in Brazil, inspired in the "American model", what Loureiro (1996, also Loureiro and Lima 1994) has defined as an incorporation of theoretical and methodological patterns practiced in advanced countries, particularly in the United States, by a part of the Brazilian research community. There is a large literature on this process, but IMPA is mostly neglected as a relevant institutional agent in the historical

transformation of Brazilian economics.[5] The aim of this paper is to contribute to the understanding of this process of internationalization, adding to the history of economics in Brazil, and the dissemination of mathematical economics. Moreover, it explores the practice of economics within a mathematics department, shedding light on the relationship between mathematics and economics from a historical and sociological point of view. It shows the impact of the establishment of a Bourbakist center of mathematics with the unintended consequence of fostering research on mathematical economics in Brazil. Furthermore, it is important to draw attention to the fact that the paper studies an ongoing process since IMPA is still an active locus of economics training. The paper draws on archival material available at IMPA[6] and different sources of secondary literature.

1. Transforming Brazilian mathematics: Nicolas Bourbaki in São Paulo

Until the interwar period, mathematics in Brazil was confined to engineering schools. Auguste Comte's positivism[7] had been a strong influence on Brazilian intellectual elite by the end of the XIX century, prompting the end of the Brazilian empire and the establishment of the republic under the rule of positivists from the army. Popular among some intellectual circles and members of the armed forces, this philosophy also had a defining influence in Brazilian mathematical education in the early XX century (da Silva 1999). Positivism endorsed the primacy of empirical methods in science and criticized the domination mathematics had on other sciences – since it was seen as pure reasoning, in contrast with sciences as physics and astronomy deemed as superior since they were based on observation (Pickering 1993, 49). As universities were almost non-existent, professional colleges of law, medicine and engineering were almost all higher education that took place in Brazil, and mathematics was limited as an instrument that engineers needed in their professional duties.

Some dissenters as Otto de Alencar Silva (1898) and Amoroso Costa (1918) rejected the positivist attitude towards mathematical reasoning, but they were the minority among higher education circles. In the thirties however, forces were shifting in Brazilian academia. In 1934, the foundation of the *Universidade de São Paulo* (USP) was an opportunity for a transformation of the traditional academic atmosphere. Local elites of

[5] Bielschowsky (1996) has no place for IMPA on his account of Brazilian economic thought – however it must be noted that by the time of publication of the book, IMPA was still a new actor in the economics conversation. Szmrecsányi and Coelho (2007) devotes several chapters on the history of multiple important economics departments in Brazil, but there is no mention to IMPA throughout the book. In another example, Klüger (2017) makes a remarkable analysis of the relations and interactions among Brazilian economists and academic and governmental institutions. Although an extensive and very complete work, IMPA barely appears in it.

[6] The author acknowledges his debt to Suely Lima, who was responsible for the preservation of the consulted archival material. She rescued the valuable material from oblivion and terrible storage conditions for her graduate studies on the memory of the institute (see Lima 2009). The author is also grateful for the collaboration of multiple IMPA's staff for access to lists of classes, students and professors.

[7] Comte's philosophy had some influence in classical political economy at the end of the XIX century; see Ekelund Jr and Olsen (1973). On Comte's philosophy, see Pickering (2003). Also, on the influence of positivism in Brazil during the XIX century, see Carvalho (1990)

São Paulo sponsored the hiring of European professors in order to organize the new institution around the *Faculdade de Filosofia, Ciências e Letras* (FFCL). Theodoro Ramos, a student of Costa and also a staunch anti-positivist, seized this opportunity to counterpoint Comte's influence in Brazilian higher education. His strategy, in line with the international objectives of the new university, was to hire Italian professors, as Luigi Fantappiè, former student of Vito Volterra, to head the reformulation of mathematics in the university. It was not done without resistance of positivists from the engineering school – several discussions at the university council were held, as positivists engineers argued that mathematicians "teach in a very abstract way, without regard for technical applications", and that engineers were more suitable to teach the discipline in an adequate note (Abdounur and Mattos 2012, 375).

Amidst this dispute, war broke up in Europe. Fantappiè, a fascist party member, returned to Italy in 1939, and as Brazil entered the war in 1942, Italians were forbidden to hold any chair at the state university. The anti-positivist group in the university administration, now led by the geneticist André Dreyfus needed an alternative. In a meeting of the administration in June 1942, the name of André Weil, a founding member of the Bourbaki group, emerged as a possible hiring (Ata da Congregação da FFCL, 05/06/1942). The job proposal would reach Weil by the help of Claude Lévi-Strauss, professor at FFCL until 1939. Lévi-Strauss introduced Dreyfus to Weil in New York, and the French mathematician arrived in São Paulo in January 1945.[8]

Bourbaki mathematics was a perfect counterpoint to positivism. If the latter upheld the primacy of the applied and the observable over pure reasoning, the former represented the opposite, "the champions of purity within the house of twentieth-century mathematics" (Weintraub and Mirowski 1994, 247). Corry (2004) has connected the Bourbaki group with contemporary mathematics identification with the concept of mathematical structure – defined by them as abstract forms, in which certain aspects of empirical reality could fit themselves into (Bourbaki 1950, 231). Bourbaki was most influent in the fields of algebra and topology, but Corry argues that "most fields of applied mathematics seem not to have been aware of or influenced by Bourbaki" (2004, 297) with the exception of mathematical economics through the work of Gerard Debreu (Ingrao and Israel 1990, Weintraub and Mirowski 1994). But not only through Debreu - in Brazil, a Bourbaki mathematical culture would take roots at FFCL and later at IMPA, where mathematical economics would also sprout.

Postwar São Paulo became a safe harbour for Bourbaki mathematicians, as Weil was just the first to join the ranks of FFCL. Oscar Zariski visited FFCL during the first semester of Weil's tenure, later remembering that Weil was the reason he had accepted to move to "such a remote place" (Parikh 2008, 79). Zariski taught algebraic geometry, Weil taught analysis and topology. During 1945 Weil came back to France for the first time since the beginning of the war to attend to a Bourbaki meeting (Weil 1992, 189-191). Shortly after that, Jean Dieudonné would be hired to work at FFCL (14/11/1945, USP Council minute). He would stay in Brazil in 1946 and 1947, where he taught algebra and topology (Anuário da FFCL 1939-49, 615-616). Weil and Diuedonné left to the United States in 1947, but

[8] Weil (1992, 185) gives his account of the encounter with Lévi-Strauss and Dreyfus, the difficulties he faced to get authorization for leaving the United States and his period in São Paulo.

Jean Delsarte came to fill the Bourbaki chair at FFCL teaching numerous courses from 1948 to 1951, as a hired or visiting professor (Anuário da FFCL 1951, 364). In 1952, four Bourbaki visited FFCL for conferences, Samuel Eilenberg, Jean Dieudonné, Laurent Schwartz, and Charles Ehresmann[9] (Anuário da FFCL 1952, 352).

The many important visiting mathematicians would provide the new generation of Brazilian mathematicians not only courses in the Bourbaki tradition, but also would connect the mostly isolated Brazilian mathematical milieu to a developing international community. *Fundação Getúlio Vargas* (FGV), a higher education foundation located in Rio de Janeiro, benefitted of the Bourbaki mathematicians at São Paulo, since they would travel across both cities to present seminars at the mathematical nucleus of the foundation. The group was headed by Lelio Gama, a veteran astronomer, who as Ramos had been a student of Costa and inherited his anti-positivist stance. The group lasted for not much time, but it was important to connect young mathematicians in Rio to the São Paulo dynamic mathematical environment. Besides the Bourbaki lectures, the FGV group received visitors from the department of Chicago, the director Marshall Stone, and professor Adrian Albert presented seminars there in 1947. The short life of the group would not stop Gama. In 1952 he became the first director of IMPA, staying at the position until 1965.

The professors who teamed up with Gama at the outset of IMPA were among the young mathematicians that gained from the presence of multiple Bourbaki in Brazil. The engineers Maurício Peixoto and Leopoldo Nachbin, who studied together at Rio de Janeiro and were also members of the FGV group, seized the opportunity offered by the sudden internationalization of Brazilian mathematics, along with fellowships opportunities available in the postwar. Nachbin received a US State Department fellowship and a Guggenheim Fellowship to work as research assistant at Chicago University from 1948 to 1950.[10] Peixoto went along to Chicago under a Rockefeller Foundation fellowship. They benefited in particular from three acquaintances made in Brazil, Weil, Stone and Albert. Nachbin's younger cousin, Paulo Ribenboim would go to Nancy in 1950-51 to study with Dieudonné and Schwartz. Luiz Henrique Jacy Monteiro was assistant to Zarinski and Dieudonné in their courses at FFCL. In 1947 he received a scholarship from the Rockefeller Foundation to study at Harvard and Chicago, coming back in 1949 (Trivizoli 2015, 55). Until the thirties, mathematics in Brazil was confined to other departments and restricted to the usefulness of its application. But, as Paul Halmos noticed in 1957 (88), "there are young men in Rio de Janeiro almost all of whose mathematical education was obtained from [Bourbaki] works". After the Bourbaki experience in the tropics, the soil was prepared for the growth of pure mathematics.

2. IMPA's first years

[9] For reports of Schwartz and Ehresmann on Brazil during their trip, see Baranyi (2016, 693-694). In his report, Schwartz classifies Leopoldo Nachbin, future founding member of IMPA as "by far the best mathematician in Brazil".

[10] Nachbin would return to Brazil in 1950 to publish his dissertation *Topologia e Ordem*. After its translation to English in 1965, and then a second edition in 1976, the work would be rediscovered by mathematical economists as Mehta (1977, 1981) and Chichilnisky (1977) – probably by the influence of Nachbin's student, José Alexandre Scheinkman.

IMPA's vocation to pure mathematics was apparent in its first years, despite its modest resources. Until 1955, the institution had just two rooms in a larger institute dedicated to physics. Even though IMPA held its headquarters in Rio de Janeiro, collaboration with FFCL in São Paulo aided institutional development – Cândido Dias, professor of mathematics at FFCL, had been one of the patrons of the foundation of IMPA through the government bureaucracy. Irving Kaplanski from the University of Chicago and Alexander Grothendieck from University of Nancy were hired as visiting professors by IMPA in 1952. However, they lived in São Paulo and used FFCL structure and resources during their time in Brazil.[11] Grothendieck stayed for another year, and George Mostow from John Hopkins University came to fill Kaplanski spot.[12] Distinct mathematicians at FFCL, as Maria Laura Mousinho and Jacy Monteiro, became associate researchers at IMPA. The institute budget was rather limited, equivalent in national currency to around seventy thousand 1954 US dollars per year in the fifties. But it was already a recognized milieu of mathematics, due to the international career of its faculty.

Lelio Gama sought to develop the non-existent applied mathematics sector of IMPA, as he exposes in annual reports to the national research council (CNPq). In 1955, he writes about his intention to hire a foreign professor to teach calculus to physics and chemistry students, following a similar strategy adopted by Costa at USP.[13] In 1957, he communicates that "something auspicious among us is the beginning of the activities in applied mathematics. Along with the formation courses in mathematics (…) to physicists and chemists, we will hold a course in game theory and linear programming, part of mathematical economics."[14] However, limited budget, modest facilities and lack of personnel would delay the development of applied mathematics and mathematical economics at IMPA.

The professor responsible for the beginnings of applied mathematics at IMPA was a twenty-three year old engineer from Rio de Janeiro, Mario Henrique Simonsen. The course held in 1958[15] turned into a published volume, *Introdução à programação linear*, containing topics on duality, the simplex method, and Koopmans's transportation model. The impact was rather small, as just 60 copies of the book circulated among few interested in mathematical economics and linear programming in Brazil during the late fifties.[16] Nonetheless, it was an updated material written in Portuguese, citing recent work in linear programming (as Koopmans 1951 and Dorfman, Samuelson and Solow 1958). Simonsen had joined IMPA in 1954 as an intern, where he learned fixed-point theorems from Peixoto's classes. After graduating from Rio de Janeiro engineering school in 1957, he became a professor in the same place, teaching engineering economics, but soon left in 1960, joining FGV as an economist. Reputable mathematicians were frequent visitors at IMPA, and a young and already recognized generation of mathematicians composed the institute's faculty, but wages and career plan at the institute were not compelling – until 1973, there was not any formal employment relationship among IMPA and researchers,

[11] Letter from Gama to CNPQ, 12/11/1952, IMPA archives
[12] Letter from Gama to CNPQ, 12/03/1953, IMPA archives
[13] Annual report to the National Research Council (CNPq), 1955, IMPA archives
[14] Ofício 44/57, Gama to CNPq, 1957, IMPA archives. Translated from portuguese by the author.
[15] One of the attendants at Simonsen's course was a young Maria da Conceição Tavares, later a renowned representant of Brazilian Keynesian economics. See Tavares, 1986, cited in Klüger 2017, 85.
[16] Ofício 108, Gama to CNPq, 08/1959, IMPA archives

only a fellowship payed by CNPq, what could explain why Simonsen chose not to stay at the institute.[17] IMPA was also very small and there was no clear perspective of growth, since the country was entering a period of financial difficulties. Simonsen had a prolific career in economic consulting for private companies and the government in the sixties, and then became minister of finance in the seventies. He never cut ties with academic circles, however, staying connected to the economics department of FGV, where he became the first head of the graduate department in 1965, leaving it in 1974 only to take on the ministry of finance. At FGV, Simonsen was a major influence to a whole generation of Brazilian economists.[18]

As Simonsen headed to a different career path, Gama needed other people to develop the applied mathematics field. However, IMPA's growth was damaged by Brazilian federal government fiscal crisis, following major public spending with the building of the new capital, Brasília, and other public works during the Kubitschek administration. After 1957, IMPA's budgets took cuts every year because of the current deficit situation on federal budget, and if in nominal terms IMPA's budget remained constant, it lost purchasing power due to the rise of inflation.[19] Gama stated to CNPq in a report that "research at IMPA in 1959 won't reach 1958 levels"[20] due to financial restrictions. The hiring of a foreign professor to develop applied mathematics that was under planning had to be cancelled.[21] In 1961, former student Djairo Figueiredo and Geraldo Ávila from São Paulo[22] came back to IMPA restarting some applied mathematics in the institute, but by the start of 1963 they had both joined the *Universidade de Brasília* (UnB) department of mathematics. The applied department at IMPA planned by Gama was struggling to develop.

Pure mathematics was better protected from the Brazilian crisis due to numerous fellowships granted by international funding. It was not without resistance however, as Baranyi (2006, 700) notes. Rockefeller Foundation deemed IMPA faculty as Elon Lima and Nachbin as essential to "the organization of advanced training of young mathematicians [in Brazil]". Nachbin had his scholarship extension denied for another year at Chicago in 1957, despite his protests that "any pioneering work is very slow and tiresome" in Rio de Janeiro. Gama wrote a letter to the Rockefeller Foundation stressing the importance of the renewal of Nachbin fellowship to IMPA,[23] but it was only through a fellowship from the Guggenheim Foundation that he could renew his time in Chicago, working along Schwartz.

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[17] IMPA is not formally associated to any university, but at the time professors as Nachbin and Peixoto held jobs at other departments since the institute could not offer much stability or remuneration. Gama was also director of the National Observatory during the time he served as IMPA director.

[18] On Mário Henrique Simonsen contributions and influence during the sixties, see Barbosa (1997); Alberti, Sarmento and Rocha (2002); Campos (1998); also Fernandez and Suprinyak (2018)

[19] Letter from Newton Ferreira Campos to Lelio Gama, 10/29/1956

[20] Ofício 6, Gama to CNPq, 01/59, IMPA archives. Translated from portuguese by the author

[21] Annual report to CNPq, 1959, IMPA

[22] Both Figueiredo and Ávila had been funded by CNPq in 1956 to study at Courant Institute, NYU. The fellowship was exclusive to applied mathematics. Figueiredo (Palis, Camacho and Lima 2003, 80) credited the initiative to Cândido Lima da Silva Dias, dean of mathematics at FFCL.

[23] Letter from Gama to Henry Miller, 20 March 1957, IMPA archives

The same happened to Elon Lima. He completed his master degree in the University of Chicago, from 1954 to 1956, under a fellowship program of the Rockefeller Foundation (Trivizoli 2015, 55), but had to resort to CNPq funds to complete his PhD at the same university. He studied algebraic topology under Edwin Spanier and had an important encounter with the post doctorate fellow Stephen Smale. As Smale headed to the Institute of Advanced Studies at Princeton in 1958,[24] Lima introduced him to another IMPA professor, Peixoto, who was spending time at the institute. This proved to be a changing event to Smale's career per his own account (Smale 1980). Smale used the transversality tool he learned from René Thom at Chicago and extended Peixoto's results on structural stability in a disk to a generalized setting. Peixoto influence led Smale to pass proposals to teach at Harvard, Chicago and Berkeley and leave to Rio de Janeiro, where he spent the last semester of his National Science Foundation post-doctorate fellowship at IMPA, in 1960. As his paycheck was cashed in dollars, the Brazilian economic crisis was a blessing in disguise, since the weakened local currency provided the Smale family with an upgraded standard of life in Rio (Batterson 2000, 50-54).

Smale had a productive time in Rio. He worked on the higher-dimensional Poincaré conjecture in topology (Smale 1960) and the horseshoe on dynamical systems, a hallmark on early chaos theory (Smale 1967). He enjoyed his mornings doing mathematics at Copacabana beach,[25] and the afternoons at IMPA discussing differential equations with Peixoto and topology with Lima (Smale 1980). The work done at IMPA helped him to receive the Fields Medal in 1966, along another former IMPA researcher, Grothendieck. With the end of his NSF fellowship in July of 1960, Smale accepted a position at Berkeley, joining other former Chicago mathematicians Spanier and Morris Hirsch. Due to the ties connecting these mathematicians to IMPA faculty, the institute enjoyed close relations with Berkeley's department of mathematics in the sixties and seventies.

The department of mathematics at Rochester University also became close to IMPA, due to Nachbin. He was visiting professor for two academic years before he was hired as a full professor in 1965.[26] Until his retirement in the early eighties, Nachbin split his time between Rochester and IMPA.

The fiscal crisis that started under Kubitschek was getting worse, transforming into a general economic and political crisis. The next president, Jânio Quadros, resigned in his first year at office, and political turbulence continued during the vice-president Goulart administration. In 1964, a right-wing military coup seized power. Intervention in academic life during the first years of the new regime changed mathematics and the future of IMPA. The UnB project encountered resistance among the military. The department of mathematics was directed by Nachbin, but as in the sixties he was sharing his time

[24] Other Chicago mathematicians as André Weil, Edwin Spanier, Morris Hirsch (another student of Spanier) and Richard Palais joined Smale at Princeton in that year, creating a small Chicagoan network of mathematicians at the university (Batterson 2000, 49)

[25] "My best-known work was done on the beaches of Rio de Janeiro, 1960!" became a famous Smale quotation among mathematicians. This was an ironic response in a letter to the NSF in 1966, after the foundation suspended his grant on the accusation that he was enjoying vacation in Europe and not performing any research duties, amidst Smale political involvement with anti-Vietnam War protests at Berkeley (see Smale to Connick, 9/16/66, Stephen Smale archive, cited in Batterson 2000)

[26] LN.T.1.002, Leopoldo Nachbin archive, MAST, Rio de Janeiro. See also University of Rochester Undergraduate Studies Official Bulletin 1964, 134.

between Rochester and Rio de Janeiro, Lima was appointed by him to coordinate the new department. However, persecution of professors in the university led three out of four professors of UnB to resign in protest against the government, including Lima, Figueiredo, and most professors from the mathematics department (Salmeron 1999). Nachbin did not joined the protest, actually helping the government to reorganize the depleted department of mathematics. After the disappointing experience at UnB, Lima visited Rochester by Nachbin's recommendation, then Berkeley for a year and returned to IMPA in 1968, along with Jacob Palis. In the following year, Manfredo do Carmo, one of the mathematicians who left UnB with Lima, moved from Berkeley to IMPA.

The expansion of IMPA in the late sixties that allowed the hiring of several professors was due to another political decision made under military rule. In 1964, the national development bank (BNDE) started a program to fund research in technology in Brazil, known as Funtec. The fund was designed for research in physics, chemistry and engineering in its first years. Lindolpho Dias, who had just undertaken the direction of IMPA, worked to include mathematics within the scope of Funtec in 1966. As the engineering school of Rio de Janeiro left its facilities at the center of the city and moved to a larger campus, IMPA used Funtec resources to reform and use part of the old building. The institute quintuplicated its facilities total area. Funtec resources also allowed IMPA to raise salaries, expand the library, hire more employees, and expand its master and PhD program in mathematics.[27] Funding profited from fast economic growth in the period known as the "economic miracle" – between 1968 and 1974, the Brazilian economy grew around ten percent per year.

3. Building networks: Araújo and Scheinkman abroad

In a period of expansion, IMPA had more space to receive students from different areas of research that were also interested in mathematics. Aloisio Araujo was a student of economics and statistics[28] when he discovered IMPA in 1966. There, he took courses under Figueiredo, who was visiting the institute in 1967, and Nachbin. In 1969 he completed his master degree in mathematics at IMPA and applied to a PhD at the United States. On Araujo's account, "there was a culture at IMPA in favour of Berkeley"[29], and with a recommendation from Lima and Nachbin, he moved to the department of statistics at Berkeley. Araujo saw statistics as a middle of the road choice among his interest in both pure mathematics and applied social science. His advisor was Lucien Le Cam, a former student of Jerzy Neyman who had been influenced by the Bourbaki tradition as a student in the University of Paris in the forties (Moore 2007, 156).

As Araujo moved to Berkeley statistics, the department also moved to Evans Hall in 1971, a new building dedicated to house mathematical sciences done at campus. Mathematics, statistics and computer science held most of the space in the new building. The departments of economics, demography, electrical engineering and journalism also received some space at Evans Hall (Moore 2007, 228-29). The offices held by the department of economics were allocated specifically to mathematical economics. In the

[27] Cardoso and Oliveira 1968, 91. See also an interview with Dias in Palis, Camacho and Lima 2003, 193.
[28] Those were two different courses in different institutions. Araujo started his course in economics at UFRJ in 1964 and statistics at *Escola Nacional de Ciências Estatísticas* in 1965.
[29] Palis, Camacho and Lima 2003, 5

department of statistics, Araujo shared the same building of Gerard Debreu, Daniel McFadden, George Akerlof and Andreu Mas-Colell. His experience at Evans Hall almost led him to pursue a second PhD in economics at Berkeley, but university regulation was not helpful, discouraging double PhDs per his own account (Palis, Camacho and Lima 2003, 8). Nevertheless, the dynamic climate of mathematical economics at Berkeley allured Araújo, as he teamed up with Mas-Colell to study general equilibrium theory.[30] They had similar trajectories: both started their college studies as undergraduate students of economics, then moved to mathematics at graduate college (Araújo at IMPA and Mas-Colell at Minnesota), and then back again to economics (or a mathematics-economics hybrid), with a detour in statistics to Araújo. During the summers, Debreu hosted seminars on mathematical economics that attracted economists from several locations to discuss general equilibrium theory.

Mathematical economics at Berkeley in the early seventies was not limited to members of the economics department. Here Smale comes back to our story. After Debreu reached him at Berkeley to discuss some mathematical problems in the early seventies, Smale got attracted to the approach of modelling an economic system that general equilibrium theory advanced. Mathematical economics was the first experience of Smale in applied mathematics – in the eighties he also made contributions to algorithms and computer science. He became a member of the advisory board of the *Journal of Mathematical Economics* at the time of its creation in 1974[31]- the first volume of the new publication had four research papers written by him. Reviewing the work of Smale in mathematical economics, Debreu (1993, 131) emphasizes the productivity of the mathematician during the period he was interested in the subject. Between 1973 and 1981, Smale published fifteen articles addressing problems "at the core of [general equilibrium] theory." At the height of this interest in mathematical economics, Smale made his second trip to IMPA, in April 1976. During the same year, he was appointed a professor of economics at Berkeley with a 0% appointment, a contract that allowed a professor to join a department without any commitment to classes in order to promote interaction and collaboration among different departments.

David Gale was a crucial example of the flexible boundaries of the departments at Berkeley. He moved to the university from Brown with a joint appointment at the department of mathematics and the Department of Industrial Engineering and Operations Research (IEOR) in 1966. As the economics department discovered that there was an effort to bring Gale to the university, a 0% appointment to the department of economics was added to the job proposal, making Gale a member of three different departments (Moore 2008, 241).

José Alexandre Scheinkman was another IMPA alumni in the Berkeley academic circle of mathematical economics, although his visits to the department were limited to the summer seminars hosted by Debreu. Scheinkman had been a student of economics in Rio in the same college of Araújo. He discovered IMPA in his first year at college, by seeing a poster announcing mathematical courses at the institute. The student had the attention

[30] Araújo and Mas-Colell collaboration at Berkeley generated a paper on the smoothness of aggregate demand. See Araújo and Mas-Colell 1978.

[31] Other members of the advisory board were Kenneth Arrow, Robert Aumann, Gerard Debreu, David Gale, Leonid Hurwicz, Edmond Malinvaud, Boris Mityagin, Roy Radner, Herbert Scarf and Lloyd Shapley.

of Nachbin, who recommended him for a scholarship from CNPq and became his dissertation advisor in the master's program, done simultaneously with the bachelor in economics. Scheinkman credits Nachbin for his first contact with mathematical economics through a book from Nikaido.[32] By the end of 1969, after obtaining his degree at IMPA, he was still unsure whether to pursue a PhD in economics or mathematics. Nachbin was a professor at Rochester at this period, and his recommendation to Lionel McKenzie led Scheinkman to the graduate program at the university. At Rochester, he could apply the mathematics he had learned at IMPA in economics, a subject that he recognized not knowing so much at the time.[33] Another influence for him at Rochester was William Brock, a former PhD in mathematics from Berkeley mentored by Gale. Scheinkman and Brock started a research together in the summer of 1973 during a workshop at Berkeley that would lead to results for optimal control systems used in growth theory (Brock and Scheinkman 1976). The mathematical dynamics he had learned at IMPA proved useful in a time that economists were aiming to build dynamical models in macroeconomics. He completed his thesis in 1972, focused on results of global stability on turnpike models with discounted utility, a topic of research developed by McKenzie in the seventies. In the published paper he acknowledges financial support from the Ford Foundation and IMPA (Scheinkman 1976).

After graduating, he left to Chicago, becoming a postdoctoral fellow. By the influence of Arnold Harbeger, the department was home for several Brazilian students during that time,[34] with whom Scheinkman interacted during the years he stayed there. He continued to attend to the seminars held during the summers of Berkeley. There he encountered the PhD in mathematics Lawrence Benveniste, another PhD student that had Gale as thesis advisor. Together they worked on an application of the envelope theorem to dynamical programming, which became known in the literature as the Benveniste-Scheinkman condition (Benveniste and Scheinkman 1979). In 1977 he got tenure at Chicago.

Araújo joined Scheinkman as a visiting professor at the department of economics at Chicago in 1978. Due to interactions at the seminars at Berkeley during the seventies, they had worked on a paper on turnpike theory (Araújo and Scheinkman 1977). From Chicago, both researchers would spend a semester at IMPA to teach courses on mathematical economics. The institute had significantly expanded the area of applied mathematics since they left at the end of the previous decade. Hosting visiting professors from foreign institutions in order to advance a specific area of research at IMPA had been a tradition, and now the institute was reenacting the strategy. During the eighties, mathematical economics would grow and disseminate at Brazil through IMPA.

4. Applying mathematics: IMPA in the seventies

With the steady economic growth Brazil was experiencing in the late sixties and the early seventies, funding restrictions was less of a problem to IMPA. The institute was deemed "one of the Brazilian research institutes with best financial resources.[35]" The enhanced

[32] Palis, Camacho and Lima 2003, 168. Scheinkman does not specifies which book, but one can suppose he is talking about Nikaido (1968)
[33] Palis, Camacho and Lima 2003, 169.
[34] Scheinkman recalls meeting Afonso Pastore, Cláudio Haddad, Fernando Holanda and Antonio Carlos Gonçalves during his time at Chicago. Palis, Camacho and Lima 2003, 173.
[35] Carlos Chagas letter to Elon Lages Lima, 21/03/68

budget supplied by the Brazilian government in better financial shape was complemented by scholarships offered by international foundations, as the Rockefeller and Ford Foundation.[36] IMPA could expand its area of activity to applied mathematics, as it was envisioned by Gama since its beginnings. Some have credited the interest of IMPA in mathematical economics to Smale's advance on the field.[37] But as soon as 1970, IMPA invited Hector Sussman, an Argentinian mathematician that obtained his PhD at Courant Institute in 1969, to teach a course on mathematical economics. However, Sussman had to cancel his visit to IMPA, and by the beginning of the decade, it was statistics the most developed applied mathematics at the institute. From the late sixties to the seventies, the federal government expanded IBGE, creating many public jobs for statisticians, what helps to explain the success of the program. The statistics program at IMPA started in 1971, and in 1972 Barry James, a PhD in statistics at Berkeley, joined the institute as professor.[38]

Mathematical economics was not let aside, despite the early success of statistics at IMPA. In a letter to Araujo in 1971,[39] the director of IMPA Lindolpho Dias writes about the beginning of an applied mathematics program, with emphasis in statistics. He also writes that a plan was underway to expand the program to include economics, two years before Smale got involved in the subject.[40] It was through the statistics program that mathematical economics slowly got into the applied mathematics program at IMPA. In 1977, the institute hired the engineer Jack Schechtman to the statistics program, expanding it to include operations research. He was a member of the IMPA community (professor of linear algebra in 1968 at the institute) that was back from Berkeley, where he completed his PhD on operational research under David Gale.

In the first semester of 1978, Schechtman offered an extra class[41] on mathematical economics. The statistics master's student Pedro Valls Pereira was the only regular student of the course.[42] In the second semester, Valls Pereira moved to London for a PhD in economics at LSE. He was back to IMPA in 1983, teaching econometrics at the institute for four years, helping the development of the mathematical economics program. In the first semester of 1979, Schechtman taught econometrics at the institute, with more success than the previous class, with eleven regular students[43]. Two engineers from ITA were

[36] William Carmichael letter to Lindolpho de Carvalho Dias, 04/09/68

[37] See, for instance, Araujo (9), Scheinkman (171, 176),

[38] In 1981, James published at IMPA an intermediate textbook on probability in portuguese that is still often used in graduate programs in Brazil (see James 2015 for the most recent edition).

[39] Letter from Dias to Araujo, 03/11/71. IMPA archives

[40] Smale had visited Brazil in 1971 to attend to a conference on dynamical systems in Salvador, organized by Palis, his former student at Berkeley. There is no evidence that Smale was already involved in mathematical economics in 1971 or that he influenced any IMPA faculty to pursue a program of economics as an applied field of mathematics at the institute.

[41] An extra class (or a *curso livre*) was a discipline with 24 hours of classes, while a full discipline had 48 hours. It was an optional course dedicated to students interested in a specific topic.

[42] As this information relies on IMPA's records on past classes, there is only information on regular students, but as flexibility was a tradition at IMPA since its beginnings there is a possibility others attended classes without being signed for the course.

[43] Other courses taught by Schechtman in his first years at the operations research program at IMPA included queueing theory and network theory. After 1980, after the start of the mathematical economics PhD course with the hiring of Araujo, Schechtman focused on the operations research program, continuing to teach in his area of expertise, programming and stochastic processes.

among the attendees of the class, and were under Schechtman supervision in their master's program on operations research, and would later pursue a PhD in economics - Armando Castelar Pinheiro, PhD in Economics at Berkeley in 1989 under the supervision of Albert Fishlow, and Ricardo Paes de Barros, PhD in Economics in 1987 at the University of Chicago, under the supervision of James Heckman. Schechtman also became thesis advisor to Marilda Sotomayor, who was pursuing a PhD in stochastic processes at PUC.[44] Schechtman presented to her a possible extension to his economic growth model developed under Gale during his PhD. Sotomayor's work led her to a postdoctoral fellowship in Berkeley. As Gale was focused at the time on matching problems, so she changed her topic of research from economic growth to game theory. In the eighties, she authored with Gale and Alvin Roth multiple papers on the subject, and an influential book, *Two-Sided Matching* (Roth and Sotomayor, 1990).

Amidst this booming interest in mathematical economics at IMPA in 1978-79, the institute hosted Araújo and Scheinkman as visiting professors. Araújo taught two courses from August to November, *Dynamical Economics* and *Microeconomics*. He has credited this period as crucial in his decision to come back for a career in Brazil in 1980, stating that he returned to Chicago "with the intention of coming back." Even though he was concerned of being isolated from the international community, he believed IMPA "created the conditions of doing science far away from the established centers" (Palis, Camacho and Lima 2003, 8-9).

Scheinkman visited Brazil from September 1978 to December 1979. He was visiting professor at IMPA and at EPGE, the graduate school of economics at FGV. Carlos Langoni, a PhD in economics at Chicago, was the director of EPGE and was responsible for the invitation to Scheinkman. At IMPA, interaction with Schechtman produced a joint work published in *The Review of Economic Studies* on general equilibrium theory under the existence of storage (Scheinkman and Schechtman, 1983). In the second semester of 1979, he taught the course *Introduction to Economic Theory* at the institute.

Economic and political events would again impact the formation of the community of Brazilian mathematical economics. The raise of US interest rates by the Federal Reserve chaired by Paul Volcker in the late seventies generated a crisis in a country with a high stock of external debt. Under these conditions, the minister of finance Mário Henrique Simonsen resigned from his position in the middle of 1979, returning to academic life as the new director of EPGE. His return allowed Scheinkman to meet the former minister for the first time. Both were participants in the *I Encontro Brasileiro de Econometria* in 1979, a meeting organized by researchers who had returned recently from a PhD in prestigious universities.[45] The meeting was fundamental to the creation of the *Sociedade*

[44]Sotomayor was a professor of mathematics at *Pontífica Universidade Católica* (PUC) at Rio, even though she had only a master degree in mathematics at IMPA. She got her doctorate degree from PUC, but was a frequent attendant to IMPA. The proximity of both universities and the flexibility of IMPA bureaucratic rules allowed her to have Schechtman as a thesis advisor.

[45] Among the organizers of the event were the members of the first SBE presidential office: Joanílio Rodolpho Teixeira (PhD Kent, 1975), Edmar Bacha (PhD Yale, 1968), Fernando de Holanda Barbosa (PhD Chicago, 1975), Roberto Macedo (PhD Harvard, 1974), José Luiz Carvalho (PhD Chicago, 1972), and Adriano Dias (PhD Vanderbilt, 1976). See Teixeira (1984, 145).

Brasileira de Econometria, a national version of the Econometric Society that became an institutional milieu of mathematical economics in Brazil.

5. Consolidation of a community: Mathematical economics at IMPA

IMPA started a formal program on mathematical economics in 1980, with the return of Araújo to the institute, now formally hired as professor. As during his first time at IMPA, he witnessed the department moving to a larger site in 1981, a three-story building located at the south zone outskirts of the Tijuca forest. He had the task of creating an economics graduate program from within a mathematics department. Despite the interest in economics that was growing in the past years through the statistics and operations research program, none of the students came from economics departments. Courses on microeconomics and econometrics were offered based on students' interest, and there was no course on macroeconomics. The program was limited to a master's degree in its first years, and it worked as a springboard for reaching a PhD placement in a recognized university in the United States. The number of brazilians doing an economics PhD abroad had grown in the seventies, due to agreements among national and foreign institutions, the majority going to universities such as Vanderbilt (Fernandez and Suprinyak, 2018), Berkeley, and Chicago (Klüger 2017, 394-441). In the eighties, doing a PhD abroad turned into a crucial passage point in order to follow an academic career (at least for a part of Brazilian research – in the early nineties, more than 80% of professors at PUC-RJ and EPGE had graduated in the United States, see Loureiro 1996, 189), or to achieve good status in the private or public sector. The advanced training in real analysis, algebra and probability theory obtained at IMPA allowed their students to have a strong application for American economics departments. This was the case for the first students to graduate in the program, Carlos Ivan Simonsen Leal and Sergio Werlang, who graduated in 1982. They had done their undergrad in engineering together at Rio, and attended some classes in mathematics at IMPA before joining the master's program in mathematical economics (Alberti, Sarmento and Rocha 2002, 240). They were also graduate students at EPGE - Leal was nephew of Mário Henrique Simonsen, director of that institution, who had a major influence on his career towards economics (Klüger 2017, 705) – but left for Princeton before acquiring their degree. Leal did his dissertation under the supervision of Harold Kuhn, while Werlang was supervised by Hugo Sonnenschein. Both done their research on game theory, graduating in 1986. Another participant in the first classes of the mathematical economics program was Paes de Barros. Although a student of the statistics program, he attended courses held by Araújo for the mathematical economics course before leaving for his PhD at Chicago.

Attracting economics students to a grad course in a mathematics department was not an easy task without establishing connections to other Brazilian economics department. IMPA is an institute without formal connections to any university, and only have graduate programs. Briefly after his return, in 1982, Araujo was invited by Simonsen to teach at EPGE, and established a close connection between IMPA and the economics department chaired by a former member of the institute. Simonsen returned to IMPA in 1984 to teach one last class in macroeconomics for the mathematicians. Araújo still holds positions in

both institutions. He recalled the academic cultural differences between IMPA and EPGE at the time:

At EPGE there were more engineering students with the ambition to work professionally with economics; they got their master's degree, had an excellent formation, and then looked for a job (...) While at IMPA students usually were pure mathematicians, without major interest in economics; they came from mathematics, and doing mathematical economics was a big step toward applications (...) It was not a competition, IMPA never joined Anpec, and the research I did was associated with pure mathematics (...) EPGE had a strong interest in inflation, macroeconomics, the big topics that Simonsen worked on. (Araújo in Palis, Camacho and Lima 2003, 12, translated from portuguese by the author)

Since then, EPGE has turned into an important department in theoretical and mathematical economics,[46] stemming from this close connection to IMPA. Leal chaired the EPGE department in 1994-97 and has been the president of FGV since 2000. The third student to enter the mathematical economics program was Paulo Klinger Monteiro, who would also play a role in EPGE theoretical turn. He was the first student to receive a doctorate degree in mathematical economics at IMPA, in 1988. Monteiro was an early exception, as the next student to receive such degree did it only in 1994, as the master's degree continued to be the regular option to study economics at IMPA. In the nineties expansion of the program, Monteiro was an important collaborator, teaching microeconomics at IMPA between 1994 and 1999, then moving to EPGE where he still holds a professor position.

The mathematical economics program was built around Araujo, but he could not underpin it solely. As it was done at FFCL during the thirties, the path to develop the graduate program was bringing visitants from the international community of mathematical economics to IMPA (see list of visiting researchers in appendix). The most frequent visitor was José Alexandre Scheinkman, almost an IMPA agent at the Chicago department. In the eighties, he visited the department during seven northern hemisphere summers. The professional networks built by Araujo during his time in the United States were helpful in order to bring visitants to his new department. It was a golden opportunity to students, as Werlang and Paes de Barro met their future thesis supervisors, Sonnenschein and Heckman, during their visits to IMPA in 1980 and 1981.

A first collaborator to the program at IMPA was the mathematician Carlos Augusto Isnard, professor of analysis. In 1982, Octavio Tourinho (PhD Berkeley in administration) and Paulo Guedes (PhD Chicago economics), researchers at IPEA, were hired to teach a class in mathematical economics for that year. In 1984, a second economist was hired to teach at IMPA. Paulo César Coutinho, returning from a PhD in the University of Pennsylvania (Costas Azariadis was his thesis advisor) taught mathematical economics

[46] To assess this claim, the author constructed an economics department ranking based on research published in Debreu's (1987) bundle of leading journals in mathematical economics – Econometrica, Review of Economics Studies, International Economic Review, Journal of Economic Theory, and Journal of Mathematical Economics – for the last ten years (2008-2018) using Tilburg University Economics Ranking sandbox. By this standard, EPGE ranks in the top 50 world list of leading departments. However, it does not appear in the top 100 list using the standard bundle of journals provided by the Tilburg Ranking.

and contract theory from 1984 to 1993, when he moved to UnB. He was thesis supervisor to six master's students during this span, including Roberto Apelfed, who left for a PhD in economics at Chicago and made a career in the banking industry. Valls Pereira and Werlang, after completing their PhD abroad, taught at IMPA in the second half of the 1980s. Werlang was thesis supervisor of Alvaro Sandroni, who continued at IMPA for a PhD under Araujo, graduating in 1994. He took an opposite path from regular Brazilian students – instead of moving to United States for graduate studies and back again to Brazil, he opted for the mathematical economics PhD at IMPA and then left to teach at Kellog School of Management (where he still holds a position).

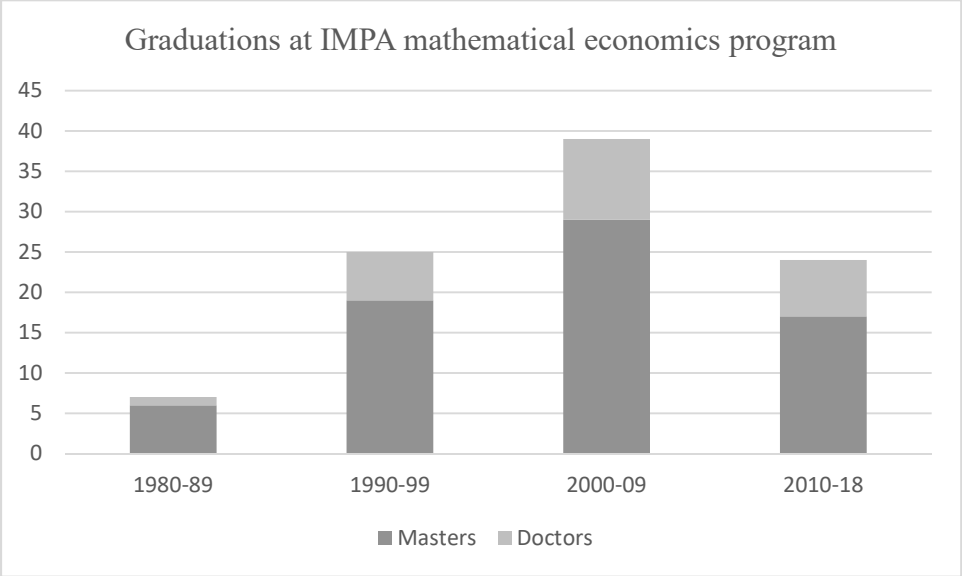
The influence of the economics program was not limited to students formally enrolled in it. The courses on econometrics attracted students from the statistics program. Other students attended courses at the institute but did not complete the degree, as Eduardo Loyo (assistant professor at Harvard, 1998-2001) who moved to the PUC-RJ master's program before his PhD in economics at Princeton. Also, the openness of IMPA to non-enrolled students attendance limits a complete assessment of students of economics looking for mathematics classes at IMPA, since they are not listed in the archives – for example, Klüger (2017, 708) cites Maria Silvia Bastos Marques[47] reporting how IMPA helped her to surpass difficulties with mathematics.

Textbooks are important elements on standardization of curricula and consolidation of a department position (Teixeira, 2014) and Araujo sought to organize his teaching at IMPA through textbooks that provided material in portuguese for students and reflected the content of his classes. In 1983 he published *Introdução à Economia Matemática*, a textbook reportedly inspired in Arrow and Hahn (1971), Varian (1978) and Ekeland (1979) – the latter being a visitant to IMPA in the same year of the publication of Araujo's textbook. The book was structured in two parts, isolated behaviour, including consumer and firm theory under certainty and uncertainty, and collective behaviour, with Arrow's impossibility theorem, general equilibrium and Pareto efficiency, and game theory. In 1993, Araujo published another textbook, *Introdução à Economia Dinâmica*, also reportedly based on Stokey and Lucas (1989). The book covered models of economic growth, as the Ramsey-Cass-Koopmans model and turnpike theory, and methods of

[47] Doctor in economics at EPGE (1987), she held several public offices, as secretary of finance of the State of Rio de Janeiro, and president of BNDES.

recursive solutions using dynamic programming. It also included an appendix on incomplete markets written by Monteiro.

Although still keeping its small size, the mathematical economics program expanded during the nineties. After a decade building status around economics circles, IMPA could attract more students interested in its program. Also, the stabilization after 1994 of the Brazilian economy that went through a period of very high inflation and public debt crisis since the eighties helped the institute (mainly maintained through public funds) on raising its resources. The nineties was also a period of internationalization of the students of the program. In that decade, 50% of graduate students (considering the master and doctorate program) came from other Latin American countries.



The placement of master's students from IMPA in PhD programs in the United States and Europe reflects the networks that the institute could build through Araujo and Scheinkman influence. An important share of the students continued their graduate studies at IMPA, mostly foreign students that came to Brazil in order to complete their doctorate at the institution. Chicago and Princeton stand out as the major destinations.

Loureiro (1996, 202) has argued the role played by invitations to participate in the brazilian government as a recognition to academic competence. Several scholars who returned to Brazil after a PhD abroad had joined the government, rarely coming back to the academic world afterwards, usually pursuing careers as consultants to the private sector. IMPA's former students had their share of influence on public policy,[48] but most continued their work at universities or research departments.

[48] Some important examples are Ricardo Paes de Barros, who had an important role in the creation of the *Bolsa-Família* program of distribution of income and Marcos Lisboa, who was secretary of economic policy at the Ministry of Finance, both serving under the presidency of Luis Inácio Lula da Silva. Another example was Araujo consultant to the reform in the Brazilian law of bankruptcy (see Araujo and Funchal 2009, and Araujo, Ferreira and Funchal 2012).

Table 1 - PhD placement of IMPA's students

IMPA	13
Princeton	4
Chicago	4
Northwestern	3
Minnesota	2
Pennsylvania	2
Total USA/Europe	26

Based on information from the Lattes platform, personal and institutional webpages, it was possible to find the current position of 53 master's graduates in the mathematical economics program (also excluding students currently in a post-doctoral position). From this group, 35 are working as professor or researchers in universities or research institutions, twenty in Brazil, nine in the United States, one in Europe and five in other Latin American countries. The permanence of IMPA's alumni within academic circles was essential to the maintenance of influence of a small department.

Concluding remarks

Brazil is a relative newcomer to the world of science, largely due to the late development of its institutions of higher education and research.[49] Mathematics had a particular path of development in the XX century. Initially limited to practical uses in correlate areas as engineering, the discipline had a turning point in Brazilian academic circles after WWII. The acceptance of the Brazilian mathematical society in the elite of the International Mathematical Union (IMU) in 2018 was a recognition of the rising internationalization and quality of mathematical work done in the country.[50] The Fields Medal bestowed to Artur Ávila (PhD at IMPA and also professor at the institute) in 2014 was a recognition of IMPA as a leading research department in dynamical systems.

The Bourbakist influence on the culture of IMPA had impact on the internationalization of the department and the research areas pursued by its researchers. And as the Bourbaki group influenced Debreu's economics, it also impacted the development of a mathematical economics program as an applied area of expertise. The graduate program in turn had a crucial role on the dissemination of mathematical economics in Brazil, by training young economists and helping to place them at renowned universities abroad, influencing other departments at EPGE through Araujo and his students, publishing

[49] This is a quotation of the opening line of the Brazilian application to group 5 of IMU (see footnote below).

[50] The IMU is a society dedicated to promote international cooperation on mathematical work, and responsible for the annual organization of ICM. IMU statute ranks adhering countries due to the importance of the research done in each place, in five groups with corresponding voting powers in the general assembly election. Other members of Group 5 (the group with most votes) are Canada, China, France, Germany, Israel, Italy, Japan, Russia, USA and UK.

textbooks on the subject, and holding several workshops, usually taught by foreign researchers visiting the institute. The history of IMPA is intertwined with the history of economics done in Brazil.

The recognition of IMPA's community of mathematical economics has not been limited to Brazilian lands. In 2006, Araujo was elected a member of the National Academy of Sciences in the Economic Sciences disciplinary section. Scheinkman also received such recognition in 2008. They are the only two Brazilian members of the NAS in economics. The paper tells the history of a mathematics department with crucial influence on the dissemination of mathematical economics within Brazilian academic circles. Furthermore, it is a history of an ongoing process, with future implications still to be seen.

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IMPA archives, Rio de Janeiro

Leopoldo Nachbin archives, MAST, Rio de Janeiro

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Appendix

List of visitants to the mathematical economics program (1980-2000)

Visitants marked with an asterisk have spent less than a month at IMPA

1979	Werner Hildebrand	Bonn
1980	Andreu Mas-Coel	Berkeley
	Hugo Sonnenschein	Princeton
	José Alexandre Scheikman	Chicago

	Fernando de Holanda Barbosa	IPEA
1981	Andreu Mas-Coel	Berkeley
	Hugo Sonnenschein	Princeton
	James Heckman	Chicago
	Jean Jacques Laffont	CRES-Univ. des Sciences Sociales
	José Alexandre Scheikman	Chicago
1982	José Alexandre Scheikman	Chicago
	Alberto Holly	Lausanne
1983	José Alexandre Scheikman	Chicago
	Ivar Ekeland	Université Dauphine
1984	Hugo Sonnenschein	Princeton
	Andrew C. Harvey	LSE
	José Alexandre Scheikman	Chicago
1985	Andrew C. Harvey	LSE
	Costas Azariadis	Pennsylvania
1986	Alberto Holly	Lausanne
	José Alexandre Scheikman	Chicago
	Peter Robinson	LSE
1987	Hugo Sonnenschein	Princeton
1988	George Box	Wisconsin
	David Bartholomew	LSE
1989	José Alexandre Scheikman	Chicago
	Mario Pascoa	Pennsylvania
1990	Alberto Holly	Lausanne
	José Alexandre Scheikman	Chicago
1991	José Alexandre Scheikman	Chicago
1992	Alberto Holly	Lausanne
	José Alexandre Scheikman	Chicago
	Mario Pascoa	Nova Lisboa
	Brigit Grodal	Copenhagen
1993	Alberto Holly	Lausanne
	Mario Pascoa	Nova Lisboa
	Dilipe Abreu*	Princeton
	Bernard Cornet*	Polytechnique
	Eduardo Engel*	Univ. Chile
	Georges Haddad*	Paris I
	Luis Quintas*	UAB (Barcelona)
1994	Alberto Holly	Lausanne
	Mario Pascoa	Nova Lisboa
	Lawrence Cristiano*	Northwestern
	David Levine*	UCLA
	Edward Prescott*	Minnesota
	Randall Wright*	Pennsylvania

1995	Alberto Holly	Lausanne
	Mario Pascoa	Nova Lisboa
1996	Alberto Holly	Lausanne
	Mario Pascoa	Nova Lisboa
1997	Alberto Holly	Lausanne
	Dieter Sondermann	Bonn
	Mario Pascoa	Nova Lisboa
	Celia Cabral	Nova Lisboa
	Pierre Yves Geoffard	ENS - Paris
1998	Alberto Holly	Lausanne
	Sylvain Sorin	Polytechnique
	Mario Pascoa	Nova Lisboa
	Celia Cabral	Nova Lisboa
	Flavio Menezes	Australian National University
1999	Mario Pascoa	Nova Lisboa
	Sudipto Bhattacharya*	LSE
2000	Alberto Holly*	Lausanne
	Zengiing Chen	Shandong