How does income distribution affect the impact of public investment on private investment? Empirical evidence from Brazil[1]

Marina Sanches (Made-FEA/USP)
Tainari Taioka (Made-FEA/USP)
Clara Brenck (Made-Cedeplar/UFMG)
Gustavo Serra (Made-Unesp)

Abstract

This paper investigates the relationship between private and public investments in Brazil. By estimating a linear Vector Autoregressive Model (VAR) for 1996-2022, we find a crowding-in effect: an increase of 10% in public investment generates a 1.48% and 2.5% rise in private investment after one and four quarters, respectively. To investigate the role of income distribution in this result, we employ a Threshold Vector Autoregressive Model (TVAR). We find the crowding-in effect only occurs in the relatively low-income inequality regime: a 10% increase in public investment results in a 0.8% and 3.4% increase in private investment after one and four quarters, respectively. Conversely, in the relatively high-inequality scenario, the response is not statistically different from zero. Thus, from a macroeconomic standpoint, diminishing inequality holds the potential to enhance the responsiveness of private investment to public investment.

Keywords: Public investment. Private investment. Income Inequality. TVAR. Crowding-in effect.

JEL Classification Codes: C32, E22, E25, H54

Área 1. ECONOMIA

[1] The authors express their gratitude to Rodrigo Orair for making available the monthly series of public investment from 1995 to 2017, whose estimation methodology is detailed in Dos Santos et al. (2012).

1. Introduction

Investment is one of the main components of aggregate demand, especially when related to long-term growth (Lavoie, 2014). Nevertheless, there is still controversy about the role of the state both as a direct investor or as an inducer of private investment. In order to fill this gap, many studies have carried out empirical estimates to capture whether public investment crowds in or crowds out private investment. Recent studies on the Brazilian economy (Alves and Luporini, 2008; Sanches and Rocha, 2010; Tadeu and Silva, 2013; Dos Santos et al., 2016; Reis et al., 2019; Bredow et al., 2022; Fraga and Ferreira-Filho, 2023; Iasco-Pereira and Duregger, 2023), found a crowding-in effect of public investment on private investment. However, the literature has shown that the magnitude of the effect depends, for example, on the target of the investment, such as strategic infrastructure sectors (Aschauer, 1989a, 1989b; Greene and Villanueva, 1991; Calderón and Servén, 2004; Perrotti and Sánchez, 2011), and economic conditions, such as periods of economic expansion and recession, and low and high credit liquidity (Conte Filho, 2013; Alves Luparini, 2008; Soave, 2016).

Inspired by these studies and motivated by recent political changes in Brazil that have the potential to affect income distribution, including the announcement of a new public investment program aiming at enabling macro-sectoral conditions for economic growth and development (Taioka et al., 2023) and a recent expansion of *Programa Bolsa Família* (Bergamin et al., 2023; Resende at al., 2023), the largest conditional cash income transfer in the world (Neves et al., 2022), we noticed a gap in the empirical literature for Brazil on the relationship between public and private investment in different economic contexts for income inequality. Understanding that the levels of income inequality generate different stimuli in the dynamism of the economy (Carvalho and Rezai, 2016), our hypothesis posits that the inequality regime may influence the magnitude of the ultimate impact of public investment on private investment. Thus, this article aims to investigate whether the effect of public investment on private investment differs in regimes of relatively high and low-income inequality.

Keynes (1964) argues for the importance of fiscal intervention in controlling the volume of investment to avoid expressive fluctuations in employment.[2] Moreover, Keynes also discusses the positive relationship between the marginal efficiency of capital (as a determinant of private investment) and the propensity to consume. Additionally, Kalecki (1945), when discussing ways of achieving full employment, argues that public investment does not exclude the need to stimulate mass consumption. In this sense, our working hypothesis is that a relatively more equal income distribution enhances the public investment multiplier, leading to a stronger crowding-in effect on private investment compared to an economic scenario with relatively higher income inequality.

Using a linear Vector Autoregressive Model (VAR) for Brazil in 1996-2022, we find a crowding-in effect, in line with the more recent literature: a 10% rise in public investment leads to a 2.5% increase in private investment after four quarters. However, using a Threshold Vector Autoregressive model (TVAR), we find that the crowding-in effect only occurs in the relatively low-inequality regime: a 10% rise in public investment leads to a 3.4% increase in private investments after four quarters. In contrast, the response in high-inequality scenarios is not statistically different from zero. Our findings underscore the significant macroeconomic implications of reducing income inequality: it has the potential to enhance the responsiveness of private investment to public investment.

The remainder of this paper is organized as follows: Sections 2 and 3 review the literature, focusing on the empirical literature about the relationship between private and public investments in the Brazilian economy. In Section 4, we elaborate on the econometric methodology and the data employed in this study. Section 5 presents the results, while Section 6 offers a discussion of them.

[2] See Seccareccia (2012) for an analysis of Keynes' proposal for the socialization of investment.

Section 7 presents some robustness tests (including control variables). Finally, in Section 8, we draw our conclusions.

2. Related literature

There is no consensus in economic theory on the relationship between government spending and private investment, with a more evident disagreement when comparing different schools of thought. Regarding public investment more specifically, some authors argue that its expansion crowds out private investment, as both types of investment might compete for physical and financial resources. In this case, the increase in public investment could raise production costs, for instance, through elevating interest rates or affecting expectations on taxation (if the public expenditure results in higher government deficit), leading to a reduction in private investment (Jacinto and Ribeiro, 1998; Cruz and Teixeira, 1999; Sonaglio et al., 2010; Reis et al., 2019).

On the other hand, the crowding-in effect occurs when private investment responds positively to public investment. The empirical literature typically presents two channels through which public investment boosts private sector investment (Bredow et al., 2022; Iasco-Pereira and Duregger, 2023; Cruz and Teixeira, 1999; Sonaglio et al., 2010; Reis et al., 2019; Jacinto and Ribeiro, 1998): i) public investment stimulates aggregate demand through its income multiplier effect, encouraging production and investment by the private sector (the so-called "accelerator effect"); ii) Public investment provides better supply conditions to the private sector (for example, through investments in infrastructure and human capital formation), contributing to increasing labor productivity and reducing production and transaction costs.

Although not exclusively, such a positive relationship between public and private investment appears mainly in demand-led growth models. Tavani and Zamparelli (2017), for instance, emphasize that the provision of public infrastructure positively affects labor productivity by providing better economic conditions for innovation. Inspired by Mazzucato (2013), the authors highlight the role of public investment in the innovative process and the productivity of the economy. In this sense, Ciaffi et al. (2024) empirically show for a group of OECD countries that an expansion in public investment related to research and development (R&D) generates a positive impact on business R&D investment.

Dutt (2013), in turn, despite constructing a demand-led growth model, considers supply constraints such as the possibility of financial crowding-out effects in a scenario where the government accumulates a public deficit. Even with such effects, however, it is possible to verify a crowding-in outcome due to the direct and indirect effects of public investment on economic activity. This positive effect may persist in the long term if productivity also responds to public investment. The author emphasizes that, even when accounting for negative expectations regarding an increase in the public deficit, crowding out is not a rule – and, in fact, it is logically plausible that public spending, especially in the form of investment, generates positive short-term and long-term effects.

The empirical literature verifies that the relationship between public and private investment differs for developed and developing economies. For instance, Soave, Gomes, and Sakurai (2016) studied the relationship between public investment growth and aggregate demand for 48 countries between 1975 and 2009. The sample was divided into two subsamples: 24 developed countries and 24 developing countries, following the World Bank classification. The results indicate a crowding-in effect in the long term. This effect is more significant for developing countries, a result that is confirmed by Izquierdo et al. (2019), who points out that the efficiency of an increase in public investment tends to be lower (higher) in countries where capital scarcity is less (more) pronounced. Moreover, when public investment is applied in strategic areas, such as infrastructure, it can enhance the positive effect on economic growth and increase the stimulus for private investment (Greene and Villanueva, 1991; Calderón and Servén, 2004).

Table 1A, in Appendix A, summarizes the main findings of the empirical literature analyzing the impact of public investment on private investment in the Brazilian context. In general, studies that investigate that relationship for years up to the 1990s report the existence of a negative relationship between public and private investment, such as Melo and Rodrigues Junior (1998), Jacinto and

Ribeiro (1998), and Cruz and Teixeira (1999) – although the latter reports a complementarity in the long term. Some other studies use data from the 1990s to the mid-2000s and have found a complementary relationship between the two types of investment (Alves and Luporini, 2008; Sanches and Rocha, 2010). An exception is Sonaglio et al. (2010), who find evidence of a crowding-out effect for the same period.

However, studies that use recent data and updated methodologies tend to find a complementary effect between public and private investments, in contrast to the evidence from previous literature (Tadeu and Silva, 2013; Dos Santos et al., 2016; Reis et al., 2019; Bredow et al., 2022; Fraga and Ferreira-Filho, 2023; Iasco-Pereira and Duregger, 2023). In particular, these more recent studies incorporate the period related to the *Programa de Aceleração do Crescimento* (PAC) - a Federal government infrastructure investment program - into the database. Furthermore, some more recent studies use, as private investment, the variable of investment in machinery and equipment (Bredow et al., 2022; Iasco-Pereira and Duregger, 2023; Alves and Luporini, 2008; Dos Santos et al., 2016) as it is the most relevant category for productivity gains and income growth (Bredow et al., 2022), and thus considered more accurate. However, Sanches and Rocha (2010) use investment (both public and private) in construction, primarily due to data availability. Finally, studies like Tadeu and Silva (2013) and Fraga and Ferreira Filho (2023) emphasize that the crowding-in effect is mainly attributed to public investment in infrastructure.

Besides the econometric literature on the relationship between private and public investments, our study is related to the empirical evidence about social benefits multipliers. The empirical literature has shown that social benefits in Brazil contribute to a substantial fiscal multiplier (Sanches and Carvalho, 2022; Resende and Pires, 2021; Orair et al., 2016) and exert positive effects on household consumption and private investment (Sanches and Carvalho, 2023), as well as to reduce income inequality (Hoffmann, 2007; Hoffmann, 2013; Souza et al., 2019). In particular, Sanches and Carvalho (2023) estimate that one unit spent on social benefits generates 2.3 units in consumption and 1.58 units in private investment after two years. In this sense, our results underline that literature, as redistributive policies, for instance, by reducing income inequality, create economic conditions wherein one would expect private investment to respond more to public investment.

3. Empirical evidence using non-linear approaches

Additionally, economic cycles are another important factor impacting the magnitude of the crowding-in effect. For instance, using an STVAR (Smooth Transition Vector Autoregressive model) for the period from 1947 to 2008, Auerbach and Gorodnichenko (2012) found significant evidence that government spending in the US has a greater impact on output in times of recession. Following a similar STVAR model, Orair et al. (2016) found comparable results for Brazil between 2002 and 2016. According to the author's findings, during economic expansions, the response of output to fiscal impulses tends to be subdued, resulting in multipliers lower than unity, with a maximum of 0.8. Conversely, in periods of economic downturns, these multipliers rise to nearly 2, signifying a more substantial impact on output during contractions in the economy.

Regarding the effect of government transfers in times of expansion and recession, using the TVAR approach (Threshold Vector Autoregressive model) for data from 2008 to 2022, Almeida et. al (2023) also verified a higher income transfer multiplier during recessions (0.51 in the short term and 0.99 in the long term), although lower than unit, in comparison to the expansion regime (0.11 and 0.31, respectively). As for the importance of credit liquidity on fiscal multipliers, Soave (2016) analyzes Brazil from 1995 to 2014 using the TVAR method, showing that, although output and income respond positively to fiscal shocks, both in times of high and low liquidity, the multipliers are higher and the responsiveness to shocks are more persistent in the liquidity-constrained regime. This indicates that in periods of low liquidity in the financial market, the fiscal shock is more effective since consumption and investment would be more tied to income and realized profit than to expected future values.

Carvalho and Rezai (2016) analyzed how low and high-income inequality regimes affect the demand regime. The authors estimated a TVAR for the US between 1967 and 2010, showing that inequality has a negative effect on output and can lead to a change in the demand regime in favor of profits. Therefore, more equitable income distributions would positively affect output. Hence, building upon the literature on crowding-in/out effects and economic cycles, in the subsequent sections we empirically address the following question: to what extent can low- and high-income inequality regimes influence the magnitude of the impact of public investment on private investment?

4. Data and Methodology

4.1. Data

We employed quarterly time series from 1996 to 2022 for the following variables:

i) Public Investment: It refers to the general government investment (comprising federal, state, and municipal governments), excluding state-owned companies. We obtained the quarterly series from the National Treasury of Brazil (STN), which provides public investment data in quarterly frequency from 2010 onward. For periods before 2010, we extracted the public investment series from the study conducted by Dos Santos et al. (2012), who estimated the series in monthly frequency. By combining both series, following the approach of Bredow et al. (2022), we achieved a comprehensive quarterly dataset. The public investment series is presented in millions of Reais, adjusted for inflation using the National Construction Cost Index (INCC-DI), from the Getulio Vargas Foundation (FGV) since public investments focus on infrastructure (Miguez, 2016), in accordance with Bredow et al. (2022). The series is also seasonally adjusted by the X-13 Arima Method[3].

ii) Private Investment: it is an index (1995=100), given in real terms, and also seasonally adjusted, calculated by the Institute of Applied Economic Research (IPEA) . As it is a monthly index, we transformed it into a quarterly frequency by taking the average of three months (as in Bredow et al., 2022). It refers to gross fixed capital formation in machinery and equipment. Similar to Bredow et al. (2022) and Iasco-Pereira and Duregger (2023), the assumption adopted here is that investments in machinery and equipment are primarily carried out by the private sector (see Dos Santos et al., 2016). The variable for investment in machinery and equipment is used by the recent literature (Bredow et al., 2022; Iasco-Pereira and Duregger, 2023; Alves and Luporini, 2008; Dos Santos et al., 2016), since it is considered the most relevant category for productivity gains and income growth (Bredow et al., 2022).

iii) Gini index for disposable income: this annual series was obtained from the Standardized World Income Inequality Database (SWIID) (Solt, 2020). It was transformed into quarterly frequency using cubic interpolation[4], following Carvalho and Rezai (2016). The Gini coefficient refers to post-tax and post-transfer income (disposable income).

iv) Installed capacity utilization of the manufacturing industry: The data were obtained from the Time Series Management System of the Central Bank of Brazil (BCB) and made available at a quarterly frequency by FGV, from their Conjunctural Survey of Manufacturing Industry.

v) Real exchange rate index (US Dollar to Brazilian Real): calculated by the Department of Statistics of the BCB (BCB-DSTAT) using the Extended National Consumer Price Index (IPCA) of the Brazilian Institute of Geography and Statistics (IBGE). We use the quarterly average of the monthly data available.

vi) Real interest rate: we use the monthly Brazilian federal funds rate (Selic rate) accumulated in annual terms (basis of 252 days) from the Open Market Operations Department of the BCB (BCB/Demab) and the 12-month percentage change in the IPCA. The data used in the estimation comprises the average of the monthly data for each quarter.

[3] Available in Eviews 12.

[4] Available in Eviews 12 ("Cubic Match Last").

vii) Primary Commodity Price Index: monthly data from the International Monetary Fund (IMF). We use the average for the quarter. The time series available starts in January 2003.

4.2. Methodology

Following the recent literature investigating the relationship between private and public investment in Brazil, we employ a vector autoregressive type of model in order to analyze the dynamic impulse-response functions derived from the empirical model (Dos Santos et al., 2016; Reis et al., 2019; Bredow et al., 2022). Before analyzing the hypothesis of this article, that is, how the level of income inequality influences the public and private investment relationship, we employ an estimation of simultaneous dynamic equations in a standard linear VAR model, as shown in Sims (1980) (see Dos Santos et al., 2016, Bredow et al., 2022), using public investment and private investment as our endogenous variables vector. Both variables were log-transformed and differenced in the first order since the Augmented Dickey-Fuller test showed they are integrated of order one. Based on the information criteria, we selected 4 lags[5].

Next, to consider a possible non-linearity in the relationship between the two endogenous variables when considering income inequality levels, we employ a Threshold Vector Autoregressive model (TVAR), as in Carvalho and Rezai (2016), Almeida et al. (2023), and Soave (2015). Following Carvalho and Rezai (2016), we utilize the Gini index as our threshold variable, enabling us to derive model results for relatively low- and high-income inequality levels.

The two-dimensional TVAR aims to estimate the non-linearity of the dynamic relationship between the endogenous variables. The threshold, or value among the possible values of the transition variable, is thus defined so that the sum of the squared residuals can be minimized, and the estimated coefficients will differ in the regimes. Tsay (1998) proposed an extension of the regime shift autoregressive model (threshold) to the multivariate context, giving rise to the TVAR. The TVAR model can be represented as follows (Almeida et. al, 2023):

$$Y_t = \left(\alpha_1 + \sum_{i=1}^{p} \beta_{1,i} Y_{t-1} \right) I[S_t \leq \theta] + \left(\alpha_2 + \sum_{i=1}^{p} \beta_{2,i} Y_{t-1} \right) I[S_t > \theta] + v_t$$

$$v_t = I[S_t \leq \theta] v_{1,t} + I[S_t > \theta] v_{2,t}$$

Where (Y_t) is a vector of endogenous variables and (S_t) is the threshold variable and θ the threshold. ($\beta_{j,i}$) is the matrix of lagged coefficients associated with period (i) and regime (j), where: $j = 1$ and $j = 2$ stand for the relatively low- and high-inequality regimes, respectively. (I) is an indicator that can be set to (1) if the condition in brackets is true or (0) if the condition is false. ($v_{i,j}$) is a vector of random errors and (α_j) is a vector of constant terms for regime (j). It should be noted that non-linearity is a property of the TVAR model, but within each regime, the model will be linear.

We performed an estimation for a two-dimensional TVAR[6] for the period 1996-2022, with public investment and private investment as endogenous variables. Again, both variables were log-transformed and differenced in the first order. We adopted one lag for the vector autoregressive model, following two of the three information criteria (BIQ and HQ)[7]. The Gini index for disposable income was utilized as the threshold variable, lagged by one period and differenced in the first order. The threshold value was automatically determined through a grid search,

[5] The criteria AIC, HQ and FPE indicated 4 lags. SC criteria indicated 1 lag. However, the estimation using 1 lag showed heterocedasticity problems.

[6] We performed the TVAR estimation using the "tsDyn" package in R.

[7] Following the parsimony principle, we chose 1 lag for the estimation, according to the criteria mentioned. The AIC criteria indicated 6 lags. Given that our sample is not very large, we opted to include one lag in order to have a higher degree of freedom. However, when we estimate the model using 6 lags, the substantial difference between the low and high inequality regimes persisted.

minimizing the sum of squared residuals[8]. We obtained linear accumulated impulse-response functions to Cholesky standard deviation innovations for each TVAR regime.

We also conduct tests for the robustness of the VAR model by including the following exogenous control variables (see Section 7): installed capacity utilization, real exchange rate, real interest rate, and primary commodity price index. All variables used were log-transformed and first-order differentiated, as the Augmented Dickey-Fuller test showed they are first-order integrated. Following three information criteria (AIC, HQ, FPE), we adopted four lags for the vector autoregressive model.

Public investment is ordered first, as it is considered the most exogenous[9] (Dos Santos et al., 2016; Bredow et al., 2022). As highlighted in the fiscal multiplier literature[10], when using high-frequency data, there is little or no fiscal policy response to unexpected shocks in aggregate demand (or components, such as private investment) within the same quarter since policymakers take more than a quarter to react to the macroeconomic conditions and decide the next steps of fiscal policy (Blanchard and Perotti, 2002; Perotti, 2007; Auerbach and Gorodnichenko, 2012; Ilzetzki et al., 2013; Orair et al., 2016; Sanches and Carvalho, 2022; Sanches and Carvalho, 2023).

5. Results

5.1. Linear VAR

Employing a standard linear bi-dimensional VAR, we estimate that an increase of 10% in public investment generates a 2.5% increase in private investment after four quarters. The immediate response is 1.48%.It is noteworthy that these results are close to the ones found by Bredow et al. (2022) (see Table 1A, in Appendix A). The response is statistically significant at the 10% level for all periods (Figure 1). Tests on the residuals of this model[11] are available in Appendix B1 and show that the estimation is stable and free of problems such as heteroscedasticity and residual autocorrelation.

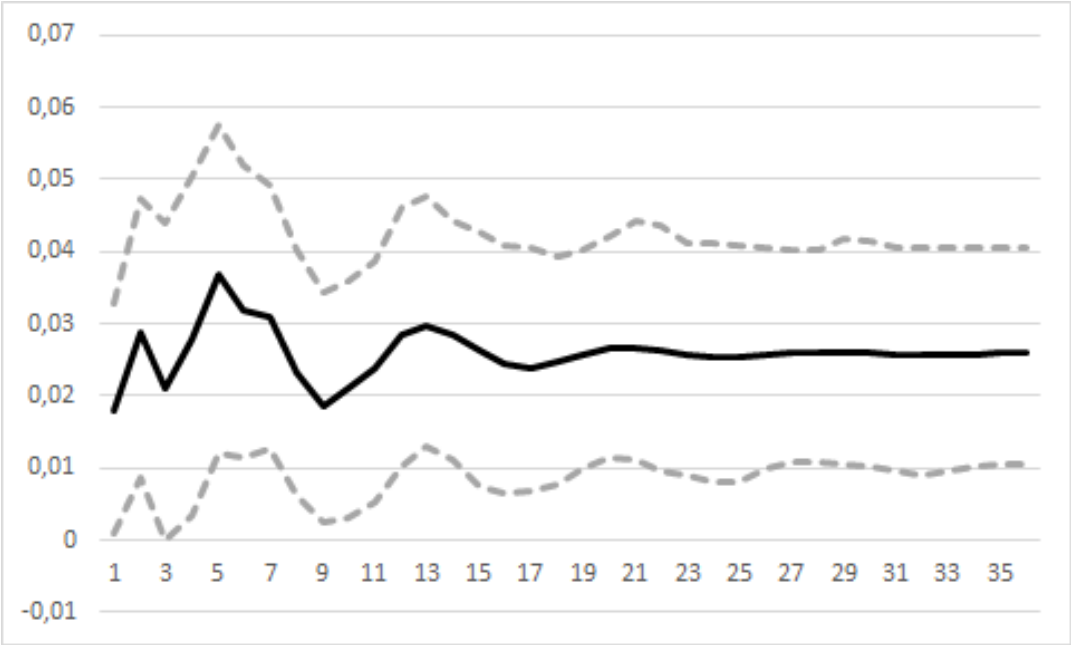
Figure 1: Accumulated response of private investment to a standard deviation shock in public investment in the linear VAR

[8] We adjusted the trimming parameter indicating the minimal percentage of observations in each regime according to the LR Test, proposed by Lo and Zivot (2001) (which is based on Hansen (1999), but for the multivariate case), and to increase the number of observations for the low-inequality regime (which has lower observations). Setting the parameter to 0.2 (Carvalho and Rezai (2016) used 0.25), the LR test reveals non-linearity, leading to the rejection of the null hypothesis of a linear model at a 5% significance level. We tested other parameter values, such as 0.15 and 0.10, for which the LR test also detects non-linearity. Our results barely changed with this variation.

[9] We also conducted estimations using an alternative specification, ordering the private investment variable first. The results remained robust to this change, with the difference between the two regimes persisting.

[10] As in the seminal paper by Blanchard and Perotti (2002) and the subsequent literature.

[11] Since the residual tests are not available in the tsDyn package in R for the TVAR model, we performed these tests only for the linear VAR models.



Source: authors' elaboration. The dashed lines correspond to confidence intervals of 90%.

5.2. TVAR

The coefficients from the TVAR estimations are presented in Table 1. Both equations for private and public investments are provided, but our analysis will primarily concentrate on the private investment equation. This focus aligns with our investigation into the crowding-in effect across two inequality regimes. The threshold parameter estimated by the method for the Gini coefficient was 0.4656, with 21.7% of the observations lower than this value ("low-inequality regime") and 78.3% of the observations higher than the threshold value ("high-inequality regime").

As indicated by the private investment equation, we observed a crowding-in effect in the relatively low-inequality regime: private investment responds positively to a shock in public investment. This impact is statistically significant at a 5% significance level, as detailed in Table 1. In contrast, within the relatively high-inequality regime, public investment does not significantly impact private investment.

Figure 2 illustrates the cumulative impulse-response function of private investment to a one-standard-deviation shock in the public investment variable for both relatively low- and high-inequality regimes. The graph underscores a notably positive response of private investment to a shock in public investment within the relatively low-inequality regime, showcasing a significant disparity between the two regimes over time, despite similar short-run responses. In other words, our findings suggest the presence of a crowding-in effect when income inequality is lower.

More precisely, the cumulative response depicted in Figure 2 reveals that a 10% increase in public investment results in a 3.4% surge in private investments after four quarters in the relatively low-inequality regime. In contrast, the corresponding response is not statistically significant in the relatively high-inequality scenario. In the short run, however, both regimes display a comparable reaction, with a 10% increase in public investment leading to an approximately 0.8% increase in private investment. Even so, as we notice in Table 1, only the immediate response of private to public investment in the relatively low-inequality regime is statistically significant.

Although it is not our focus in this article, Table 1 shows a positive response of public investment to private investment for the relatively high-inequality regime and a negative in the case of relatively low-inequality. Appendix C shows the impulse response functions for this exercise.

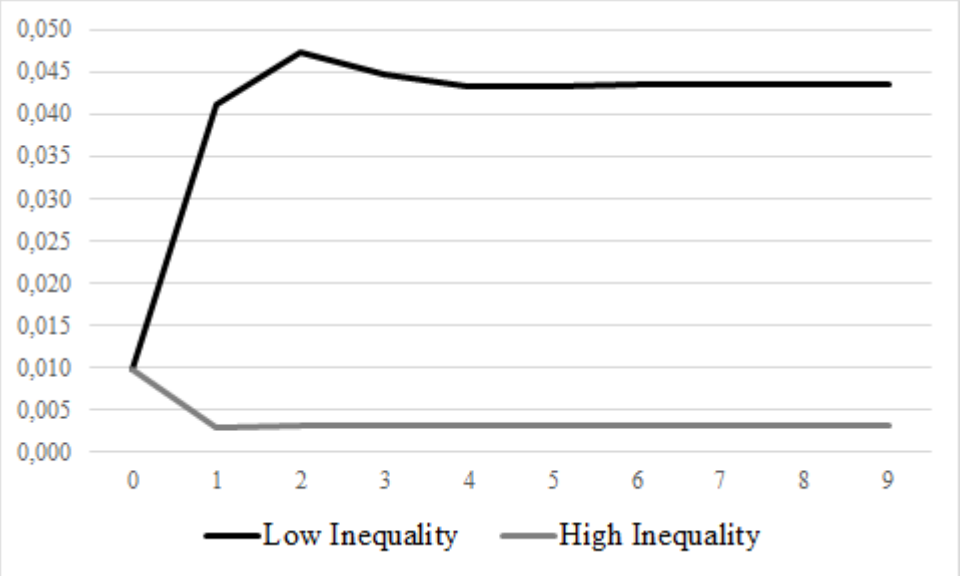
Table 1: Results of the two-dimensional TVAR estimation for Brazil (1996-2022) using the Gini index for disposable income as a threshold

	Private investment equation	Public investment equation

	Low inequality	High inequality	Low inequality	High inequality
private investment (-1)	0.0281	-0.0731	-0.4822**	0.4349***
public investment (-1)	0.2477**	-0.0480	0.2041	0.0345
intercept	-0.0294	0.0160	-0.0258	0.0026

Notes: Significance: ***1%, **5%, *10%.
Source: authors' elaboration.

Figure 2: Accumulated response of private investment to a standard deviation shock in public investment in relatively low- and high-inequality regimes



Source: authors' elaboration.

6. Discussion

Our findings are consistent with recent literature on the Brazilian economy, revealing a crowding-in effect. Importantly, our results parallel those of Bredow et al. (2022), who employed a linear VAR model for Brazil using similar time series data from 1996 to 2018. While their study found that a 10% expansion in public investment leads to a 2.04% increase in private investment over time (after four quarters), our analysis reveals a 2.5% increase for the same period of time. Moreover, Bredow et al. (2022) find an immediate effect (in the first quarter) of 1.64%. Our results for the linear VAR indicate a similar impact (of 1.48%).

When allowing for different regimes using the TVAR approach, we find that the crowding-in result only appears in the relatively low-inequality regime: a 10% expansion in public investment leads to a 0.8% and 3.43% increase in private investment after one and four quarters, respectively. Conversely, in the relatively high-inequality scenario, the response is not statistically different from zero.

It should also be noted that the most recent literature, which includes in the sample the sharp decline in the Gini index period in the mid-2000s, has found positive effects of public investment on private investment (see Table A1 in Appendix A). Therefore, our result of the crowding-in effect being more pronounced in the relatively low-inequality regime aligns with and reinforces the findings in that literature, underscoring a positive relationship between reduced inequality and investment.

In the relatively low inequality scenario, lower-income brackets, exhibiting a higher inclination to consume (as observed in Palomo et al., 2022, for Brazil), contribute to a more dynamic economy since there is a redistribution from the class with a higher propensity to save to the class with a higher propensity to consume (Carvalho and Rezai, 2016; Kalecki, 1942; Kalecki, 1952). This dynamic setting not only stimulates household consumption but also drives private investment, which responds positively to increased aggregate demand, commonly referred to as the accelerator effect (Hein and Vogel, 2008; Onaran and Galanis, 2012; Naspad and Storm, 2007; Stockhammer et al., 2009; Stockhammer and Stehrer, 2011).

A surge in public investment, for example, produces a significant multiplier effect, typically higher than one, as evidenced in Brazil (refer to Sanches and Carvalho, 2022; Orair et al., 2016; Pires, 2014; Resende and Pires, 2021; Castelo Branco et al., 2017). According to our findings, this economic stimulus positively impacts private investment, particularly in scenarios where inequality is lower, fostering a more dynamic aggregate demand environment. In such circumstances, the impetus for investment is heightened, driven by the increased dynamism in consumption stemming from a more equitably distributed income.

Our results indicate that the crowding-in effect only appears in the relatively low-inequality regime, which indicates the importance of redistributive policies to mitigate income inequality and enhance the responsiveness of private to public investment. This aligns with empirical findings in the literature highlighting the relevance of the social benefits multiplier effect. Since social benefits are targeted towards individuals in lower-income groups, who exhibit a higher propensity to consume, their multiplier effect is relevant, making the economy more dynamic (Cardoso et al., 2023; Sanches and Carvalho, 2023; Resende and Pires, 2021; Orair et al., 2016). This evidence may contribute to elucidating our finding that private investment responds significantly to the stimulus in public investment (crowding-in effect) when income inequality is lower.

7. Robustness check

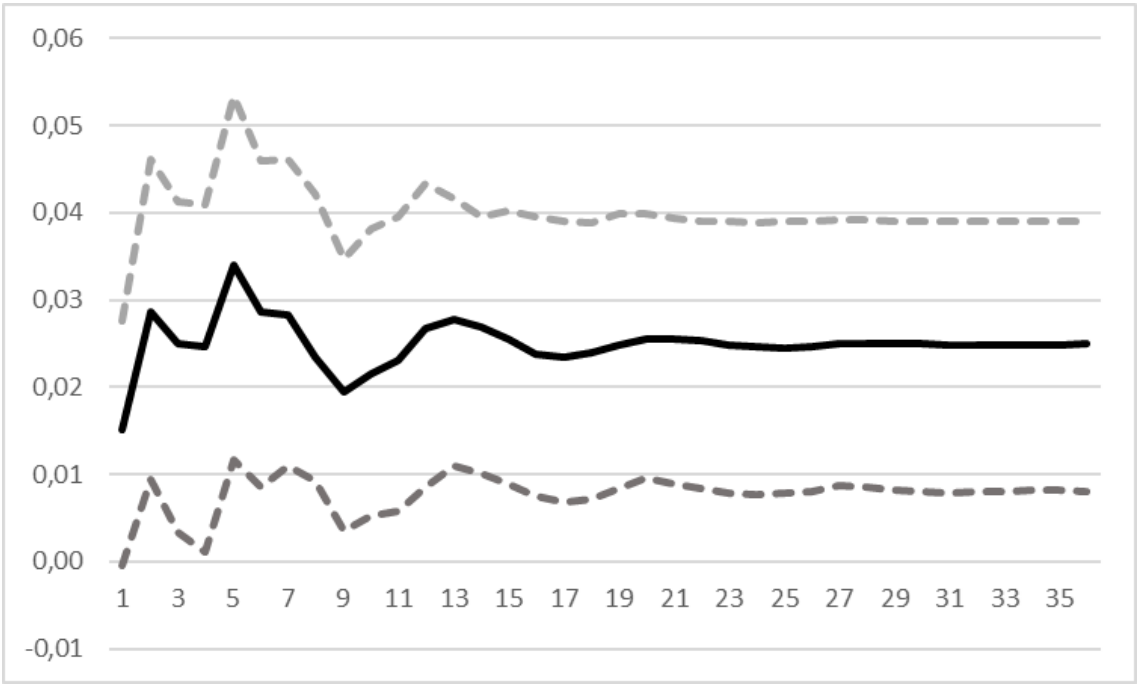
7.1) Control variables

In this section, we check the influence of control variables[12]. We introduce additional control variables based on relevant literature. Firstly, we consider the possibility of private investment being influenced by aggregate demand, known as the accelerator effect (as discussed in section 2.3). To account for this, we incorporate a control variable representing the degree of capacity utilization, serving as a proxy for aggregate demand (Fraga and Ferreira-Filho, 2023; Reis et al., 2019; Bredow et al., 2022; Alves and Luporini, 2008; Jacinto and Ribeiro, 1998).

Our analysis indicates that this variable is positive and statistically significant at the 0.1% level, implying a positive impact on private investment. Figure 3 shows that the crowding-in effect still appears when we control for aggregate demand: a 10% increase in public investment leads to an increase in private investment of 1.24% in the first period, and of 2.21% after four quarters.

Figure 3: Accumulated response of private investment to a standard deviation shock in public investment in the linear VAR using the capacity utilization variable as exogenous

[12] Since the possibility of including control variables is not available in the tsDyn package in R for the TVAR model, we have included controls only for the linear VAR model.



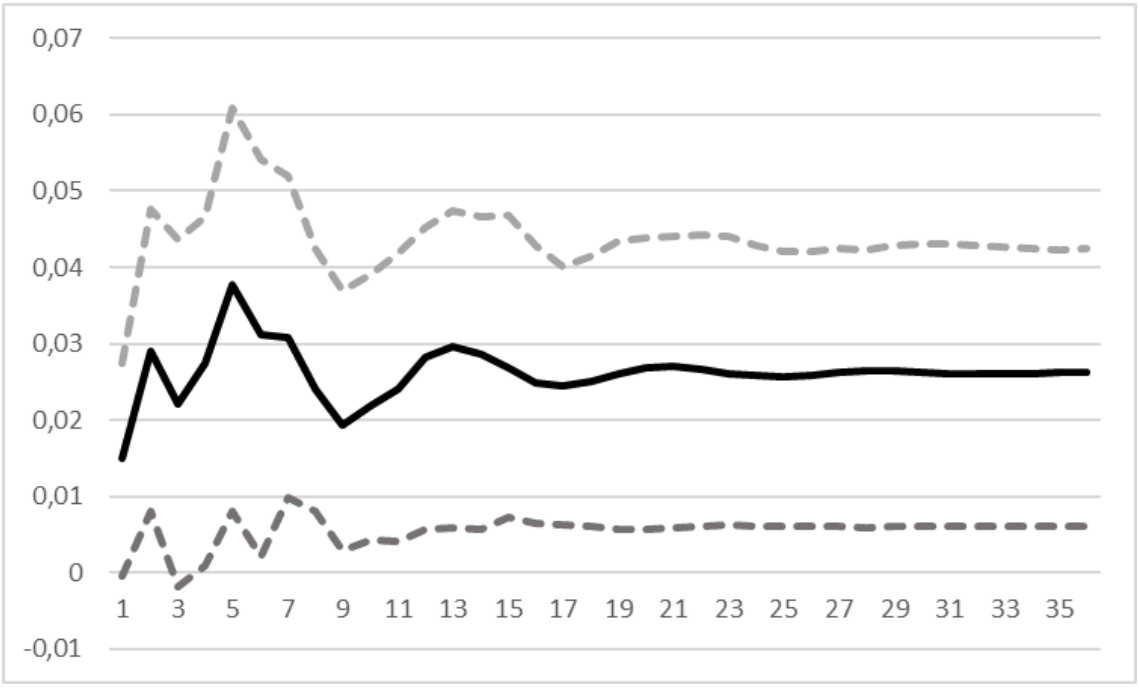
Source: authors' elaboration. The dashed lines correspond to confidence intervals of 90%.

This finding aligns with previous studies by Jacinto and Ribeiro (1998), Alves and Luporini (2008), Sonaglio et al. (2010), and Reis et al. (2019), which have similarly observed a positive relationship between capacity utilization and private investment. The rationale behind this relationship is that companies tend to increase their investments as the utilization of installed capacity rises. Moreover, in the presence of demand stimuli, private investment tends to rise until it achieves the desired capital stock (Conte Filho, 2013).

Furthermore, we have included the exogenous real exchange rate variable (Ribeiro and Teixeira, 2001; Tadeu and Silva, 2013; Reis et al., 2019; Bredow et al., 2022). We found a negative and significant effect at the 5% level (Tadeu and Silva, 2013; Dos Santos et al., 2016). According to Figure 4, with the inclusion of the exchange rate variable, a 10% increase in public investment implies an increase in private investment of 1.25% and 2.44% in the first and after four quarters, respectively.

The negative sign of the exchange rate may indicate that higher accurate (i.e. devaluated) exchange rates may not encourage imports of capital goods and, therefore, reduce private investment. This result is in line with Ribeiro and Teixeira (2001) and Tadeu and Silva, 2013), who found that the exchange rate significantly and negatively affects private investment in Brazil.

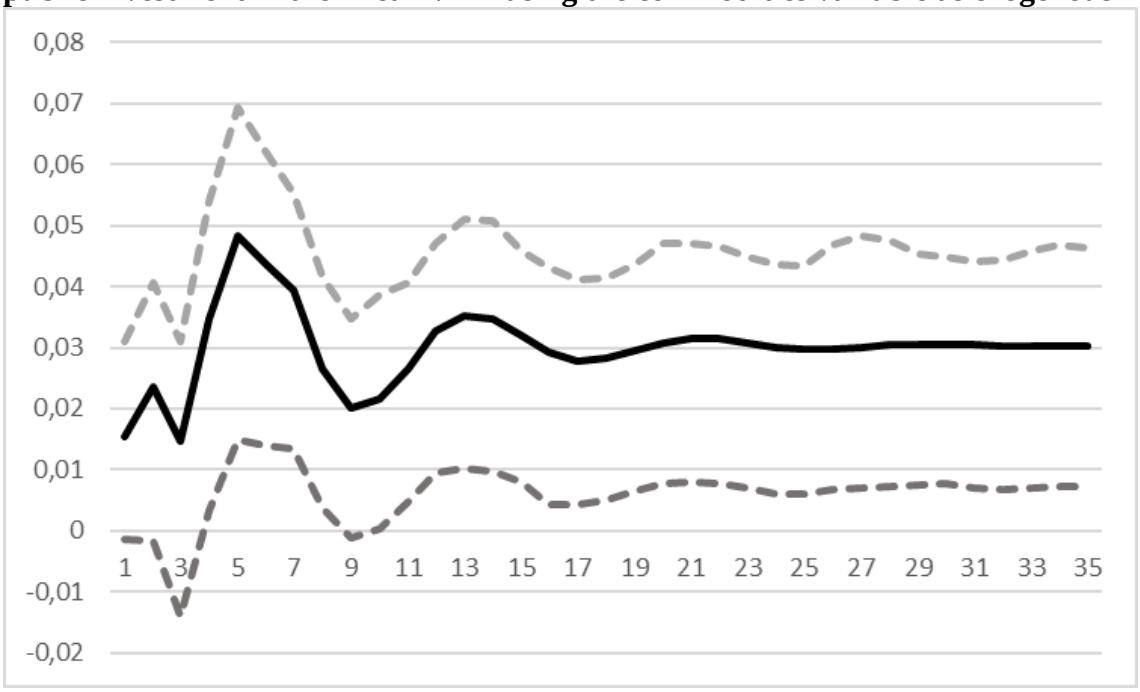
Figure 4: Accumulated response of private investment to a standard deviation shock in public investment in the linear VAR using the exchange rate variable as exogenous



Source: authors' elaboration. The dashed lines correspond to confidence intervals of 90%.

We have also included the commodity price index[13] as a control variable. As in Dos Santos et al. (2016), the exogenous variable commodities had a positive effect and was significant at the 1% level. This effect may be associated with increased agricultural and extractive investment participation. Another explanation would be indirect channels, for example, the reduction of external restrictions on economic growth and the expansion of the domestic consumer market (Dos Santos et al., 2016). Figure 5 shows that our crowding-in result is robust to the inclusion of the commodity price index as a control variable: a 10% increase in public investment leads to an increase in private investment of 1.23% and 2.97% after one and four quarters.

Figure 5: Accumulated response of private investment to a standard deviation shock in public investment in the linear VAR using the commodities variable as exogenous



Source: authors' elaboration. The dashed lines correspond to confidence intervals of 90%.

[13] This exercise is performed for the period 2003-2022. Data for the commodity prince index are available by the IMF from 2003 onwards.

Finally, we have included the interest rate control variable (Bredow et al., 2022; Alves and Luporini, 2008; Cruz and Teixeira, 1999). This control variable did not show statistical significance. Notably, in addition to not being significant, the interest rate has a negative value, in line with Bredow et al. (2022). It is worth remembering one of the arguments why public investment would generate a crowding-out effect on private investment. If there were an increase in the interest rate to finance public investment through the sale of public bonds, the availability of private-sector credit would be reduced. Moreover, private investment would consequently be discouraged. Our result suggests the opposite since public spending did not exert pressure on prices and costs in the economy. Furthermore, it had a potential negative impact on private investments. Therefore, public investment would not compete with private investment for resources by raising the interest rate or increasing production costs (putting pressure on inflation). On the contrary, public investment would generate a crowding-in effect on private investment due to its multiplier effect and the physical expansion and expansion of the domestic market. In addition to acting as a facilitator of services and reducing production costs of private investment, increasing the productivity of the economy, mainly when aimed at meeting infrastructure demands (Aschauer, 1989a and 1989b; Greene and Villanueva, 1991; Calderón and Servén, 2004 and Perrotti and Sánchez, 2011; Reis et al., 2019).

We observe that the findings from Section 5 remain robust even after including control variables: the response of private investment to a shock in public investment continues to be statistically significant at 10% across most periods (Figures 3, 4, and 5). Additionally, tests on the residuals of the models in this Section are available in Appendix B2 and indicate that the estimations are stable and do not exhibit heteroscedasticity or residual autocorrelation.

7.2) Alternative methodology: non-linear local projections

We used the local projections model based on Jordà (2005) to perform the robustness test of the model. The non-linear version of this methodology uses a smooth transition function as in Auerbach and Gorodnichenko (2012) to separate the data into two regimes. Unlike the TVAR model, the local projections package in R allow us to estimate the confidence interval for the two inequality regimes[14].

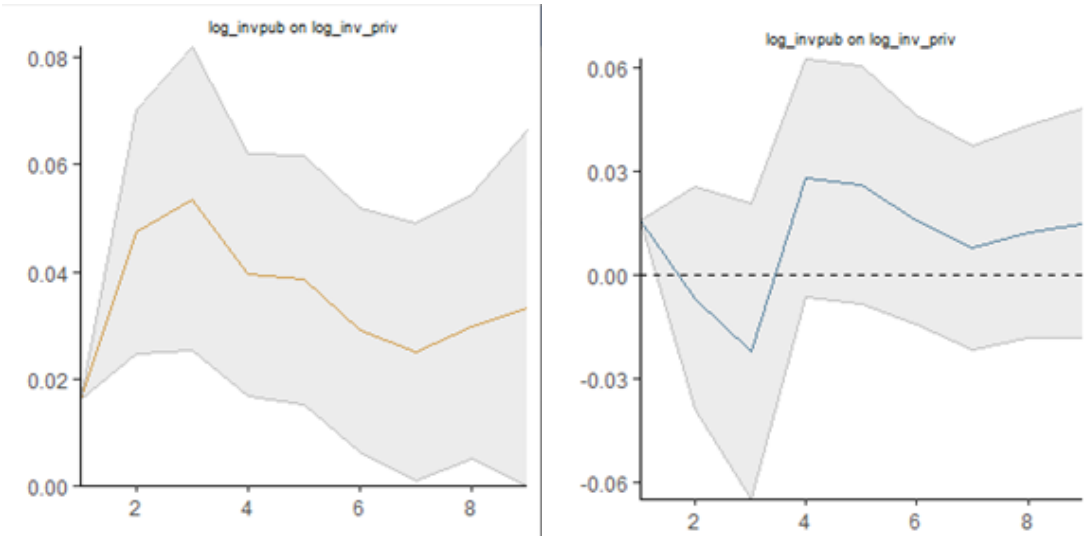
Figure 6 shows that our main conclusions still hold in this case. The local projections methodology produces results that are similar to the TVAR's ones. The impact was positive and significant for both regimes in the first period. After the second period, the impulse-response function for the low inequality regime was significant at 10% throughout the entire period after the shock, until period 8, while it was not significant for the high inequality regime[15].

Figure 6: Responses of private investment to a standard deviation shock in public investment - high and low inequality, respectively

Low inequality **High inequality**

[14] The package 'pirfs'.

[15] The switching function is similar to the work by Auerbach and Gorodnichenko, 2012. We used gamma as 2, but our conclusions are robust to other values. We used the lags for the model chosen by the AIC criteria.



Source: authors' elaboration. The gray lines correspond to confidence intervals of 90%.

8. Concluding remarks

This paper has analyzed how relatively high- and low-inequality regimes impact the magnitude of the response of private investment to public investment. Our findings support the conclusions drawn from recent literature, indicating that public investment has a crowding-in effect on private investment—that is, the latter responds positively to the former.

More specifically, our linear VAR baseline estimation concludes that an increase of 10% in public investment generates a 2.5% rise in private investment after four quarters. Using a TVAR model, we have demonstrated that this crowding-in effect only occurs in the relatively low-inequality regime: a 10% increase in public investment results in a 3.4% increase in private investment after four quarters. On the other hand, in the relatively high-inequality regime, this response is not statistically different from zero.

Our results suggest that the state has a key role as a driver of investment and that its role is crucial in implementing and coordinating investment programs (as in the case of the New *Programa de Acelaração do Crescimento* - PAC in Brazil). Furthermore, programs aimed at greater income distribution are crucial to improving the welfare of low-income populations and boosting the effects of macroeconomic stimuli, as public investment is more effective at stimulating private investment in a less unequal economy.

In alignment with the literature on fiscal multipliers of social benefits, our results underscore the substantial macroeconomic potential of policies designed to mitigate inequalities. In this context, these policies play a pivotal role in advancing social equity and propelling more inclusive economic growth.

References

ALMEIDA, E. S.; RIBEIRO, E. P.; SCHOMMER, S. Transferências fiscais de renda, gastos e seus efeitos multiplicadores na atividade econômica brasileira. Anpec, 2023.

ALVES, J.; LUPORINI, V. Determinantes do investimento privado no Brasil: Uma análise de painel setorial. Anais do 36o Encontro Nacional de Economia – Anpec, 2008.

ASCHAUER, D. A. "Is government spending productive?," Journal of Monetary Economics, Vol. 23, pp. 177-200, 1989a.

ASCHAUER, D. A. "Does public capital crowd out private capital?," Journal of Monetary Economics, Vol. 24, pp. 171-188, 1989b.

AUERBACH, A. J.; GORODNICHENKO, Y. Measuring the output responses to fiscal policy. American Economic Journal: Economic Policy, v. 4, n. 2, p. 1-27, 2012.

BERGAMIN, J.; SERRA, G.; SANCHES, M.; GOMES, J.; NASSIF-PIRES, L. Quais os possíveis impactos do Novo Bolsa Família? Uma análise sobre os efeitos do NBF nos índices de pobreza e extrema pobreza e sobre o PIB do Brasil. Nota de Política Econômica n. 44, Made USP, 2023.

BLANCHARD, O.; PEROTTI, R. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. Quarterly Journal of Economics, v. 117, n. 4, p. 1.329-1.368, 2002.

BREDOW, S.; CUNHA, A.; LÉLIS, M. Investimentos públicos e privados no Brasil entre 1996 e 2018. Revista de Economia Política, v.42, n.4, p.957-976, 2022.

BRENCK, C. Wage inequality and employment composition in Brazil: a VEC estimation for the period 2004-2019. Nova Economia, v.31, n.2, p.345-380, 2021.

CALDERÓN, C.; SERVÉN, L. The effects of infrastructure development on growth and income distribution. Central Bank of Chile, 2004. (Working Paper, n. 270).

CASTELO-BRANCO, M. A.; LIMA, E. R; PAULA, L. F. Mudanças de regime e multiplicadores fiscais no Brasil em 1999-2012: uma avaliação empírica com uso da metodologia MS-SBVAR. Pesquisa e Planejamento Econômico, v. 47, n. 2, p. 7-62, 2017.

CARDOSO, D.; CARVALHO, L.; LIMA, G. T.; NASSIF-PIRES, L.; RUGITSKY, F.; SANCHES, M. The multiplier effects of government expenditures on social protection: A multi-country study.Working Paper nº 18, Made/USP, 2023.

CARVALHO, L.; REZAI, A. Personal income inequality and aggregate demand. Cambridge Journal of Economics, v.40, n.2, p. 491-505, 2016.

CIAFFI, G.; DELEIDI, M.; MAZZUCATO, M. Measuring the macroeconomic responses to public investment in innovation: evidence from OECD countries. Industrial and Corporate Change, 00, p. 1-20, 2024.

CRUZ, B.; TEIXEIRA, J. The Impact of Public Investment on Private Investment in Brazil, 1947-1990. Cepal Review, n. 67, p. 75-84, 1999.

DOS SANTOS, C.; ORAIR, R.; GOBETTI, S.; FERREIRA, A.; ROCHA, W.; SILVA, H.; BRITTO, J. Estimativas mensais da formação bruta de capital fixo pública no Brasil (2002-2010). Economia Aplicada, v. 16, n. 3, p. 445-473, 2012.

DOS SANTOS, C.; MODENESI, A.; SQUEFF, G.; VASCONCELOS, L.; MORA, M.; FERNANDES, T.; MORAES, T.; SUMMA, R.; BRAGA, J. Revisitando a dinâmica trimestral do investimento no Brasil: 1996-2012. Revista de Economia Política, v. 36, n.1 (142), p. 190-213, 2016.

DUTT, A. K.. Government spending, aggregate demand, and economic growth. Review of Keynesian Economics, 1(1), 105-119. 2013

FRAGA, J.; FERREIRA-FILHO, H. The effects of infrastructure and public investment on the elasticity of private investment: an empirical investigation for Brazil. Brazilian Journal of Political Economy, v. 43, n. 1, p. 275-298, 2023.

GREENE, J.; VILLANUEVA, D. Private investment in developing countries: an empirical analysis. IMF Staff Papers, v. 38, n. 1, 1991.

HANSEN, B. Testing for linearity. Journal of Economic Surveys, v. 13, n. 5, pp. 551-576, 1999.

HEIN, E.; VOGEL, L. Distribution and growth reconsidered: empirical results for six OECD countries, Cambridge Journal of Economics, v. 32, n. 3, p.479–511, 2008.

HOFFMANN, R. Transferências de renda e redução da desigualdade no Brasil e em cinco regiões, entre 1997 e 2005. In: BARROS, R. P.; FOGUEL, M. N.; ULYSSEA, G. (Org.). Desigualdade de renda no Brasil: uma análise da queda recente, v. 2. p. 17-40. Brasília: IPEA, 2007.

HOFFMANN, R. Transferências de renda e desigualdade no Brasil (1995-2011). In: CAMPELLO, T.; NERI, M. (Org.). Programa Bolsa Família: uma década de inclusão e cidadania,. p. 207-216. Brasília: IPEA, 2013.

IASCO-PEREIRA, H.; DUREGGER, R. Public investment, infrastructure, and private investment in Brazil: is there a crowding-in effect? Anais do 51o Encontro Nacional de Economia – Anpec, 2023.

ILZETZKI, E.; MENDOZA, E.; VÉGH, C. How Big (Small?) Are Fiscal Multipliers? Journal of Monetary Economics, v. 60, p. 239-254, 2013.

IZQUIERDO, A; LAMA, R; MEDINA, J. P.; PUIG, J.; RIERA-CRICHTON, D.; GRAMONT, C. A. V.; VULETIN, G. J. Is the Public Investment Multiplier Higher in Developing Countries? An Empirical Exploration. Working Paper No. 289, 2019.

JACINTO, P.; RIBEIRO, E. Co-integração, efeitos crowding-in e crowding-out entre investimento público e privado no Brasil: 1973-1989. Teoria e Evidência Econômica, v. 6, n. 11, p. 143-156, 1998.

KALECKI, M. A Theory of profits. The Economic Journal, v. 52, n. 206/207, p. 258, 1942.

KALECKI, M. Full Employment by Stimulating Private Investment? Oxford Economic Papers, n. 7, pp. 83-92, 1945.

KALECKI, M. Theory of Economic Dynamics. Routledge, 1952 [2007].

KEYNES, J. M. The General Theory of Employment, Interest and Money. London: Macmillan & Co. Ltd., 1964.

LAVOIE, M. Post-Keynesian Economics: new foundations. Cheltenham: Edward Elgar, 2014.

LO, M.C.; ZIVOT, E. Threshold Cointegration and Nonlinear Adjustment to the Law of One Price. Macroeconomic Dynamics, Cambridge University Press, v. 5(4), pp. 533-76, 2001.

MAZZUCATO, M. The Entrepreneurial State, London and New York: Anthem Press, 2013.

MIGUEZ, T. Evolução da Formação Bruta de Capital Fixo na Economia Brasileira 2000-2013: uma análise multissetorial a partir das matrizes de Absorção de Investimento (MAIS). Ph.D. Dissertation. Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2016.

MELO, G.; RODRIGUES JÚNIOR, W. Determinantes do investimento privado no Brasil: 1970-1995. Texto para discussão n. 605, IPEA, 1998.

NAASTEPAD, C. W.; STORM, S. OECD demand regimes (1960–2000). Journal of Post Keynesian Economics, v. 29, n. 2, p.211–46, 2007.

NEVES, J.; VASCONCELOS, F.; MACHADO, M.; RECINE, E.; GARCIA, G.; MEDEIROS, M. The Brazilian cash transfer program (Bolsa Família): a tool for reducing inequalities and achieving social rights in Brazil. Global Public Health, v.17, n.1, p.26-42, 2022.

ONARAN, Ö.; GALANIS, G. Is aggregate demand wage-led or profit-led? National and global effects. Conditions of Work and Employment Branch. International Labour Organization, 2012.

ORAIR, R. O.; SIQUEIRA, F. de F.; GOBETTI, S. W. Política Fiscal e Ciclo Econômico: uma análise baseada em multiplicadores do gasto público. XXI Prêmio Tesouro Nacional, 2016.

PALOMO, T.; CARVALHO, L.; TONETO, R. Marginal propensity to consume heterogeneity and redistributive policies: The Brazilian case. Working Paper, n. 010. Made/FEA-USP, 2022.

PEROTTI, R. In search of the transmission mechanism of fiscal policy. NBER Macroeconomics Annual, v. 22, 2007.

PERROTI, D.; SANCHEZ, R. J. "La brecha de infraestructura en América Latina y el Caribe", Serie DRNI N°153, United Nations ECLAC, Santiago do Chile, 2011.

PIRES, M. Política fiscal e ciclos econômicos no Brasil. Economia Aplicada, v. 18, n. 1, p. 69-90, 2014.

REIS, C.; ARAÚJO, E.; GONZALES, E. Public investment boosted private investment in Brazil between 1982 and 2013. Journal of Economic Issues, v. 53, n. 3, p. 813-840, 2019.

RESENDE, C.; PIRES, M. O impulso de multiplicador fiscal: implementação e evidência para o Brasil. Estudos Econômicos, v.51, n.2, p.213-243, 2021.

RESENDE, A.; GOMES, J.; SERRA, G.; NASSIF-PIRES, L. Do Bolsa Família ao Brasil sem miséria? Duas décadas de luta pela universalização da cidadania. Nota de Política Econômica n. 43, Made USP, 2023.

SANCHES, M.; CARVALHO, L. A contribuição da política fiscal para a crise brasileira de 2015-2016: uma análise baseada em multiplicadores de despesas e receitas primárias do governo central no período 1997-2018. Nova Economia, v. 32, n. 1, p. 07–36, 2022.

SANCHES, M.; CARVALHO, L. B. Multiplier effects of social protection: a SVAR approach for Brazil. International Review of Applied Economics, v.23, n.1, p. 93-112, 2023.

SANCHES, N.; ROCHA, F. Investimentos estaduais públicos e privados: "bens" substitutos ou complementares? Economia Aplicada, v.14, n.2, p. 211-223, 2010.

SECCARECCIA, M. The role of public investment as principal macroeconomic tool to promote long-term growth: Keynes's legacy. *International Journal of Political Economy*, 40(4), pp. 62-82, 2012.

SIMS, C.A. Macro Economics and Reality. Econometrica, 48, 1-48, 1980.
SOAVE, G. P. Choques Fiscais e Instabilidade Financeira no Brasil: Uma Abordagem TVAR. ANPEC, 2016.
SOAVE, G. P.; GOMES, F. A. R; SUKURAI, S. N. Efeitos do gasto do governo sobre o consumo privado: evidências de países desenvolvidos e em desenvolvimento. Estudos Econômicos, v. 46, n. 1, p. 5-41, 2016.
SOLT, F. Measuring Income Inequality Across Countries and Over Time: The Standardized World Income Inequality Database. *Social Science Quarterly* 101(3):1183-1199. SWIID Version 9.6, 2020.
SONAGLIO, C.; BRAGA, M.; CAMPOS, A. Investimento público e privado no Brasil: evidências dos efeitos crowding-in e crowding-out no período 1995-2006. Revista Economia, v. 11, n.2, p.383-401, 2010.

SOUZA, P. H.; OSORIO, R.; PAIVA, L.; SOARES, S. Os efeitos do Programa Bolsa Família sobre a pobreza e a desigualdade: um balanço dos primeiros quinze anos. IPEA. Texto para Discussão n. 2499, 2019.

STOCKHAMMER, E.; ONARAN, O.; EDERER, S. Functional income distribution and aggregate demand in the Euro area. Cambridge Journal of Economics, n.33, p.139–159, 2009.

STOCKHAMMER, E.; STEHRER, R. Goodwin or Kalecki in Demand? Functional Income Distribution and Aggregate Demand in the Short Run. Review of Radical Political Economics, p.1–17, 2011.

TADEU, H.; SILVA, J. The Determinants of the Long Term Private Investment in Brazil: An Empirical Analysis Using Cross-section and a Monte Carlo Simulation. Journal of Economics, Finance and Administrative Science, v. 18 (special issue), p.11-17, 2013.

TAIOKA, T.; SANCHES, M.; BRENCK, C.; OLIVEIRA, R. O Novo Programa de Aceleração do Crescimento (PAC): comparação com os PACs anteriores e estimação de seu efeito no PIB. Nota de Política Econômica n.45, Made USP, 2023.

TAVANI, D.; ZAMPARELLI, L. Government spending composition, aggregate demand, growth, and distribution. Review of Keynesian Economics, v.5, n.2, p.239-258, 2017.
TSAY, R. S. Testing and modeling multivariate threshold models, Journal of the American Statistical Association, 1998, vol. 93, no. 443, 1188–202.

Appendix A:

Since our focus is on investigating possible crowding-in or crowding-out effects, we have relegated the "main result" column in Table 1A to this finding. Other results, however, have also been reported in the "additional results" column, but they are not the focus of this article.

Table 1A: Summary of the empirical literature on the relationship between public and private investment in Brazil

Study	Period	Methodology	Main Result	Additional Results
Melo and Rodrigue	1970-1995	Error correction model	**Crowding-out effect:** Public investment displaces around	-The main factor affecting private investment (positively) is demand.

s Junior (1998)			a third of private investment in the short term.	-Interest rates and inflation rates have a negative impact on private investment.
Jacinto and Ribeiro (1998)	1973-1989	First difference regression (short term)	**Crowding-out effect*:*** private investment responds negatively to public investment.	-The degree of capacity utilization has a positive effect on private investment. -Credit (BNDES) has no statistically significant impact.
Cruz and Teixeira (1999)	1948-1990	ADL (*Autoregressive Distributed Lag*)	**Crowding-in effect in the long term**: public investment positively impacts private investment in the regression that considers the long-term effect. **Crowding-out effect in the short term**: public investment in the current period affects private investment negatively..	-Expected demand is the main factor in determining private investment (accelerator effect) -Interest rates have no significant impact.
Alves and Luporini (2008)	1996-2005	Panel regression for industrial sectors	**Crowding-in effect*:*** -Positive effect of public investment on the variable "acquisitions of fixed assets in industrial sectors".	-The level of activity in the sector, the level of capacity utilization, and the availability of BNDES credit positively affect investment. -The real exchange rate and external indebtedness have a negative impact, while the interest rate has an impact close to zero.
Sonaglio et al. (2010)	1995-2006	VECM (*Vector Error Correction Model*) – long term	**Crowding-out effect:** -A 1% increase in public investment reduces private investment by 0.429%.	- Private investment responds positively to demand, and negatively to the tax burden, the capital price, and the interest rate.
Sanches and Rocha (2010)	1991-2004	Panel regression for Brazilian states.	**Crowding-in effect:** - A 1% increase in public investment raises private investment by between 0.74% and 1.135%.	-Positive effect of demand on private investment. - Interest rates have a negative impact on private investment.
Tadeu and Silva (2013)	1996-2011	Regression analysis	**Crowding-in effect*:*** -A 1% increase in public investment in infrastructure raises private investment by 0.113%.	-Positive effect of demand on private investment. -A 1% increase in public investment not related to infrastructure has a negative impact of 0.0741% on private investment. -Interest rates have no significant effect, while the exchange rate has a negative impact on private investment. - BNDES credit to the private sector has a positive impact.

Dos Santos et al. (2015)	1996-2013	VAR (*Vector Autoregressive Model*)	**Crowding-in effect*:*** -Private investment responds positively to public investment.	-There was a positive relationship between commodity prices and private investment and a negative between the exchange rate and private investment.
Reis et al. (2019)	1982-2013	VECM (*Vector Error Correction Model*) – long term	**Crowding-in effect*:*** -Private investment responds positively to public investment	-Private investment responds positively to demand, the degree of capacity utilization, the share of profits in income, and the real exchange rate. -Industrial production responds positively to public investment.
Bredow et al. (2022)	1996-2018	VAR (*Vector Autoregressive Model*)	**Crowding-in effect*:*** -A 10% increase in public investment generates a 1.64% increase in private investment in the first period. -The cumulative effect over time is 2.04% after four quarters and 1.89% after twenty quarters.	-No response from the economy's prices to expansions in public investment (exchange rate, Selic interest rate, and machine prices do not respond significantly).
Fraga and Ferreira-Filho (2023)	1960-2013	*Bayesian Model Averaging* e *Weighted-Average Least-Squares*	**Crowding-in effect*:*** -Private investment responds positively to public investment, especially in infrastructure.	-Private investment responds positively to demand, to the real exchange rate and the availability of credit, and negatively to the real interest rate.
Iasco-Pereira and Duregger (2023)	1947-2017	ARDL (*Distributed Lag Autoregressive Model*)	**Crowding-in effect*:*** -A 1% increase in public investment generates an increase of approximately 0.6% in private investment (in the short term and long term). - A 1% increase in infrastructure investment generates an increase of between 0.6 and 0,7% in private investment in the long term and 0.5% in the short term.	-Aggregate demand and the terms of trade have a positive effect on private investment. -No robust evidence of the impact of inflation, exchange rates, BNDES credit, and labor costs.

Source: author's elaboration based on the literature.

Appendix B: Residual tests for the Linear VAR estimations

B.1. Tests for the Linear VAR presented in Section 5

Based on the information criteria, we selected 4 lags. The residual test for autocorrelation (Box-Jung test) showed a p-value of 0.78 (lag 1), 0.95 (lag 2), 0.85 (lag 3), 0.92 (lag 4), 0.86 (lag 5), 0.92 (lag 6), 0.83 (lag 7), 0.58 (lag 8), indicating that they do not have autocorrelation. The ARCH test for heteroscedasticity showed a p-value of 0.1674, indicating that the model is free of this problem. Furthermore, the model is stable since all the eigenvalues of the coefficient matrix are

smaller than one. Finally, the normality test Jarque Bera rejects the null hypothesis that residuals are normally distributed[16] .

B2. Tests for the Linear VARs presented in Section 7

-Linear VAR using the degree of capacity utilization as a control variable

Based on the information criteria, we selected 4 lags. The residual autocorrelation test (Box-Jung test) presented p-values of 0.69 (lag 1), 0.92 (lag 2), 0.87 (lag 3), 0.94 (lag 4), 0, 87 (lag 5), 0.93 (lag 6), 0.85 (lag 7), 0.59 (lag 8), indicating that they do not have autocorrelation. The ARCH test for heteroscedasticity presented a p-value of 0.80, indicating that the model is free from this problem. The model is stable, as all eigenvalues in the coefficient matrix are less than one. Finally, the Jarque Bera normality test rejects the null hypothesis that the residuals are normally distributed.

-Linear VAR using exchange rate as a control variable

Based on the information criteria, we selected 4 lags. The residual autocorrelation test (Box-Jung test) presented p-values of 0.71 (lag 1), 0.89 (lag 2), 0.87 (lag 3), 0.95 (lag 4), 0, 90 (lag 5), 0.94 (lag 6), 0.87 (lag 7), 0.62 (lag 8), indicating that they do not have autocorrelation. The ARCH test for heteroscedasticity presented a p-value of 0.77, indicating that the model is free from this problem. The model is stable, as all eigenvalues in the coefficient matrix are less than one. Finally, the Jarque Bera normality test rejects the null hypothesis that the residuals are normally distributed.

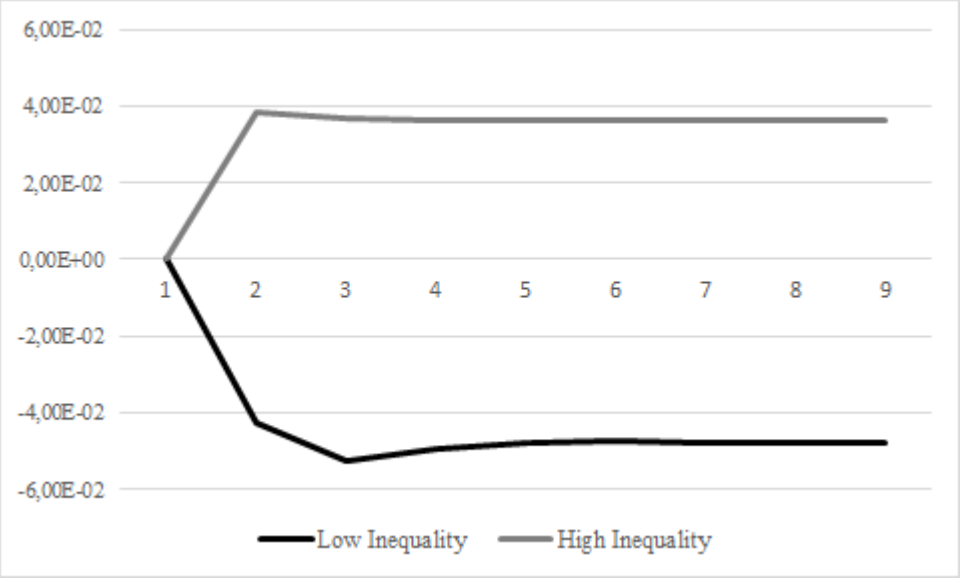
-Linear VAR using commodity price index as a control variable

Based on the information criteria, we selected 4 lags. The residual autocorrelation test (Box-Jung test) presented p-values of 0.74 (lag 1), 0.93 (lag 2), 0.90 (lag 3), 0.95 (lag 4), 0, 85 (lag 5), 0.92 (lag 6), 0.72 (lag 7), 0.39 (lag 8), indicating that they do not have autocorrelation. The ARCH test for heteroscedasticity presented a p-value of 0.05, indicating that the model is free from this problem. The model is stable, as all eigenvalues in the coefficient matrix are less than one. Finally, the Jarque Bera normality test rejects the null hypothesis that the residuals are normally distributed.

Appendix C

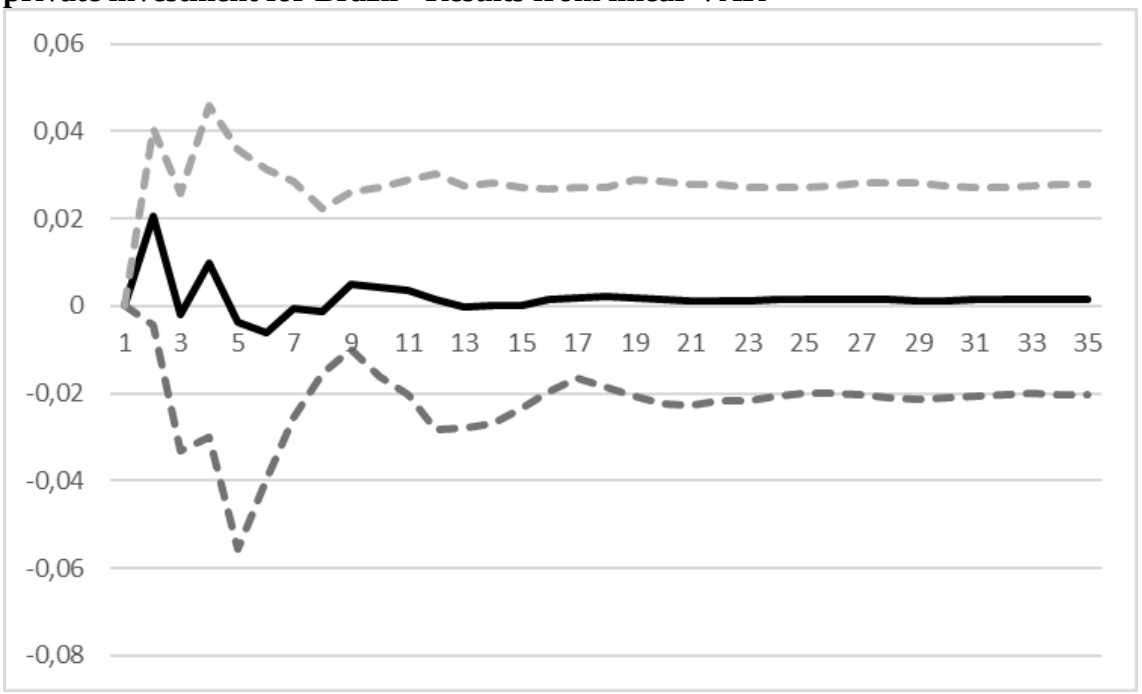
This Appendix shows the results for the impact of private investment on public investment. While the TVAR estimations show statistically significant results - see Table 1 in Section 5 (Figure 1C), the linear VAR approach (Figure 2C) does not show a statistically significant impact.

Figure 1C: Accumulated response of public investment to a standard deviation shock in private investment in the relatively low- and high-inequality regimes - Results from TVAR



[16] Normality is hard to achieve in the short run time series. As pointed out by Brenck (2021, p.378), "The Law of Large Number states that when sample size tends to infinity, the sample mean converges to the population mean, and the error term becomes normally distributed so that the lack of normality can be due to shorter term data".

Figure 2C: Accumulated response of public investment to a standard deviation shock in private investment for Brazil - Results from linear VAR



Source: authors' elaboration. The dashed lines correspond to confidence intervals of 90%.