BALANCING SHORT-RUN COSTS AND LONG-RUN GAINS: THE WELFARE IMPLICATIONS OF INVESTING IN NEW TECHNOLOGIES IN DEVELOPING COUNTRIES

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Abstract

There is a long tradition in the economic literature that recognizes learning and the diffusion of new ideas and technologies as one of major drivers for growth, especially in developing countries. However, while adopting a new technology mainly involves cost, some technologies may also be human and physical specific, i.e., vintage specific. The introduction of a new capital good may require on-the-job training or fresh investment in the educational system or even physical changes on the assembly line. This paper presents an AK model with embodied capital technology, i.e., new ideas or technologies embodied in capital goods, so the country must invest in order to have access to the new technology. To capture these features the paper has employed a Nelson-Phelps catch-up equation in an AK growth model. This model presents some very compelling dynamics: a) the possibility of catching-up and leapfrogging in an AK model structure, and b) the prospect of a negative growth and non-monotonic transition toward a balanced growth path, due to the adoption cost. The optimal pace of adopting technology generates a trade-off between short-run costs and long-run benefits. These results are of particular interest for policy makers in developing countries, given that some benefits from technology adoption may only appear after the economy learns more about the new technology. Countries may be faced with a choice, whereby the more complex the technology, the higher the short-run costs. However, the long-run gains will also be higher.

Key-words: Embodied technological progress, adoption cost, learning, AK growth model.

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1. Introduction

Understanding economic growth as a dynamic process is essential for comprehending the varying behaviors that have been exhibited by countries throughout history. Two key elements in the recent success stories of economic catch-up are learning and adoption. More than a quarter of the disparity in GDP per capita among countries may be attributed to the adoption and diffusion of technologies (Comin and Hobijn, 2010). Many economic thinkers, from Schumpeterian and evolutionary economists, such as Nelson and Winter (1982), to more mainstream authors such as Parente and Prescott (1994) or Parente (1994), have emphasized the pivotal role of adoption in enabling lagging countries to catch up.

In most cases, countries do not invent new technologies but rather, have to adopt existing ones. Introducing these technologies, even those that have become well-established, may bring about a significant impact on the growth of both developing and developed nations, since the adoption process involves costs. One illustrative example of an incentive to reduce the cost of adopting new technologies may be observed in the semiconductor industry, known as the Mead-Conway revolution. In the late 1970s, the development of new chips within company labs was prohibitively expensive. The design and architecture of new chips were inseparable from the assembly of a blueprint, thereby necessitating simultaneous execution. This structural limitation posed a significant threat to the development of the semiconductor industry and its ability to keep pace with Moore's Law, which aimed to exponentially increase computer capacity. To address this challenge, the Defense Advanced Research Projects Agency (DARPA) initiated the Very Large Scale Integration (VLSI) project, thus enabling designers to develop new architectures independently from chip production. While a few companies created VLSI projects, DARPA identified that the widespread adoption of VLSI among academia and industry players, rather than being confined to a few select firms, would reduce the cost of developing new chip designs and improve production efficiency. This move revolutionized the semiconductor industry, thus enabling a surge in new technologies and architectures, along with the development of software tools for more efficient circuit design.

Although DARPA did not invent VLSI technology, its strategic approach demonstrated how public policy could reduce adoption costs in the semiconductor industry and enhance technological advancement. Furthermore, it helped bridge the gap between designing new chips and implementing them on production assembly lines (Van Atta et al., 1990; Miller, 2023). [2]

[2] Another case illustrating how adoption costs may influence growth and productivity is the example of Intel, which designed the primary chips for IBM and PCs, known as the X-86 architecture, and became a dominant technology, especially when paired with Microsoft products. However, this architecture was not ideal for emerging technologies, such as mobile phones and artificial intelligence. While Intel explored alternatives such as reduced instruction set computer (RISC) architecture, competing in new sectors would have required a substantial effort and possibly the abandonment of profitable X-86 products (Milller,2023; Allworth, 2020).

These examples confirm the proposal by Jovanovic (1997), in which the main engine of growth depends upon the decision on whether to adopt existing technologies. He claimed that this aspect has been underemphasized in growth modeling. The message from this literature is that the adoption of new technologies is essentially costly, those features must be embedded in a model on learning and diffusion as being key variables in catching up. Jovanovic (1997) estimated that in the US, the loss in GDP due to technology adoption is around 10%. In addition, the adoption process may be 20 times more costly than innovation. It is therefore possible to suppose that this issue is of even greater significance in developing countries.

Adopting new technology implies that the timing involved in introducing a new technology varies across firms and countries. First, some technologies may be human and physical specific, i.e., vintage specific. The introduction of a new capital good may require on-the-job training or new investment in the educational system or physical changes on the assembly line. These elements, plus the fact that technological progress is embodied in machines, could explain some of the productivity slowdown observed in economies after adopting a new technology. The well-established argument in the literature is that adopting new ideas is costly, and that furthermore, it takes time for an innovation to diffuse throughout an economy. Recent research on the relationship between negative growth and the introduction of information and communication technology (ICT) has demonstrated a significant initial reduction in production, reaching as much as 28%. However, as the economy learns about and diffuses this technology, long-term growth may increase by 18% (Ayerst, 2022).

Greenwood and Jovanovic (2001) noted that incorporating adoption costs and learning curves into vintage models could enhance the explanations on productivity slowdown. However, despite extensive research into new technologies, few studies have explored the dynamic relationship between adoption costs, learning and diffusion, and embodied technological progress. The model presented in this paper addresses these issues by illustrating how negative growth and nonlinear dynamics in optimal growth rates are able to occur. The model also facilitates a comparison of different types of interventions for designing development policies aimed at fostering growth and catch-up processes.

Moreover, in recent years, industrial policies have regained the attention of policymakers. Modern industrial policies explicitly target economic structural transformation in order to stimulate innovation, productivity, and economic growth. The model presented herein provides multiple instruments for boosting growth and bridging technological gaps in developing countries. Governments may consider subsidizing capital accumulation, promoting the adoption of more complex technologies, reducing adoption costs (as seen in the DARPA example), or accelerating the diffusion of new technologies. In addition, the structure of the model enables the welfare analysis of each instrument to be evaluated, thereby allowing governments to craft a mix of policies to foster growth and development. [3]

[3] For an excellent review of the modern literature on industrial policies, see Juhász, Lane and Rodrik (2023).

The model addresses a gap in the discussion on adopting technology in economies that need to learn and diffuse technology, particularly when technology is embodied in new machines. Technically, we have utilized embodied technology in an *AK* endogenous growth model and a Nelson-Phelps catch-up equation to demonstrate that the inclusion of adoption costs may result in negative growth (Nelson and Phelps, 1966, Abramovitz, 1986). The *AK* structure enables us to fully describe the welfare and link it to policy variables. The model generates intriguing transitional dynamics and displays leapfrogging possibilities, depending on the technology and learning parameters. The convergence of output growth toward long-run growth may even exhibit non-monotonic behavior when there is a steep learning and diffusion curve. The model enables an explicit path for the capital and consumption dynamics, thereby facilitating welfare analysis.

The policy implications suggest that, as with Schumpeterian models, learning and diffusion may be significant drivers of growth and enhance welfare, especially in developing economies. Adopting a new technology may take time to translate into productivity gains, ultimately contributing to long-run growth and welfare, despite any initial delays. Additionally, the level of complexity of the adopted technology may also impact welfare. A U-shaped relationship exists between technological complexity and welfare, in which very simple technology results in lower long-run growth but with lower adoption costs. Thus, economies face a trade-off between higher long-run growth and a short-run reduction in growth and may face even negative growth.

The paper is organized as follows: the next section presents a literature review, illustrating how the model may fill some of the gaps observed in the current literature on growth. Section 3 presents the model and the balanced growth path, while the following section reveals the conditions for negative growth and the relationship of welfare function with learning and diffusion, as well as welfare and the level of technological complexity. Lastly, section 5 presents the possibility of leapfrogging and catching up.

2. Middle income trap and growth strategies

One of the major concerns relating to growth for developing economies exists in countries that have faced the so-called middle income trap. Following a certain successful history regarding growth, countries that could have moved on from a very low level of income, became stuck at this intermediate income level. The World Bank and other institutions have recognized this fact and claim that there has been a "practice gap" between the Solow and endogenous growth models.[4] While the former is useful for addressing growth issues and shaping policies in low-income countries, its key feature, the exogeneity of technology, is a drawback for discussing the prospects of middle-income countries. While endogenous growth models delve into technology, they are more focused on creating new technology for advanced economies than on helping middle-income countries adapt and diffuse technology so as to catch up. (Gills and Kharas, 2015).

[4] A particularly good reference along these lines is Acemoglu et. al. (2006), even though their focus moves toward the political economy of development and growth strategies.

The main proposal of this paper is to address this issue, by building a simple growth model that is able to focus on the adoption and diffusion of new technology in an embodied technology framework. Learning and the slow diffusion from the frontier to laggard firms have been used in the endogenous literature to explain the reduced growth in developed economies (Akcigit and Ates, 2021). Recent, robust empirical evidence has indicated that lower level of learning and the diffusion of new technologies as being partially responsible for reducing productivity growth among OECD countries. For example, firms have faced significant costs in order to access digitalization, and the slowdown in diffusion rates has played an important role in explaining the global reduction of growth. Furthermore, the literature has also demonstrated that even if policy makers stimulate the expansion of technological frontiers, diffusion to lagging firms will not necessarily be automatic. In conclusion, it is possible that intervention to promote diffusion and learning in the economy, may enhance productivity, and reduce the barriers to access new technologies (Criscuolo et. al., 2015, 2016). Ferraz et. al. (2020) estimated the cost and the willingness to adopt new technologies and digitalization in the Brazilian economy, and demonstrated that digitalization in Brazil moves at a very slow pace, and that a very small number of firms have plans to climb the technological ladder. There is an extensive literature on learning curves and the costly adoption of new technologies. Bahk and Ghort (1993) estimated a learning curve and reported that productivity gains are about 15% over the first fourteen years after implementing new technology. An excellent discussion on the implications of learning and diffusion may be also found in Stiglitz and Greenwald (2015). Learning and diffusion appear to be an important part of the picture and are linked to successful strategies for growth.

A supply chain disruption allied with “premature deindustrialization” constitute recent additional major concerns for international economies. Those two facts have initiated a debate on the possibility of a resurgence of industrial policy. As mentioned above, new empirical evidence has demonstrated that policies aiming at structural change may be effective. (Juhász, Lane and Rodrik, 2023). The debate on policy however, has not been concerned with whether a country should design industrial policies, in a broad sense, but rather how this should be done. The present model aims to contribute to this discussion, and despite its very simple structure, it nevertheless provides the possibility for many public interventions, with a mix of policies and effective welfare impact.

Another debate regarding the recent lack of growth in Latin America countries, in comparison to countries in East Asia, concerned the relatively low investment rate affecting a steady, persistent growth in the GDP, thereby leaving the middle income trap. Some policy responses have been aimed at stimulating capital accumulation. Nonetheless, there has been substantial debate regarding the lack of positive impacts on productivity following an investment boom in the 2010s, shortly after the 2008 financial crisis. Some studies reported either nonsignificant or even negative relationships between investment and firm-level productivity (e.g., Messa, 2015; Vasconcelos, 2017). The growing literature has also indicated the misallocation of capital, that is to say, incentives and policies that boost low-productive firms or sectors. Inefficient firms or sectors have been artificially kept running in the economy due to the wrong incentives. These facts would explain the low productivity growth. Even though this may have occurred, in this paper, the model adopts a second possibility, i.e., that negative growth may be an optimal response in a developing economy, when the country faces adoption costs. Even if the economy faces negative growth, the process of learning and diffusion leads the economy to a higher future growth. As clamed in Restuccia and Rogerson (2017), the discussion on misallocation needs to be improved in order to achieve a dynamical effect. In short, this paper contributes to the discussion on growth strategies, and demonstrates that an economy may present a drop in the level of consumption and capital as an optimal response to the process of adopting technology.

3. Adoption Cost, Embodied Technology and AK model

The model used in this study is an *AK* model with a central planner that maximizes a constant relative risk aversion (CRRA) utility function, represented by a logarithmic function for simplicity. The use of the *AK* model, which employs capital in a broader sense, facilitates an explicit welfare analysis, provides a comprehensive view of the dynamics of consumption and capital, and presents the entire economic transition. Consequently, policy implications and the choices of new interventions become much clearer in these models.[5]

It is assumed that a gap exists between leading countries and a developing economy, where the former is at the technological frontier while the latter lags behind on the technological ladder. In accordance with the literature, it is possible to infer the presence of a technological gap in developing countries. This phenomenon, as outlined by Greenwood and Jovanovic (2001), implies the existence of a disparity in technology adoption, hence developing countries are located below the technological frontier. Furthermore, this gap expands in proportion to the complexity or to the 'amount to be learned.' In essence, a given developing country not only lags behind in the comprehensive utilization of technology, but also experiences an increasing divergence or distance from optimal use as technological complexity grows. Parameter $A(t)$ is employed to quantify and assess the current deviation from the technological frontier which, for simplicity, is normalized to 1. Parameter q represents the complexity of the technology adopted by the country and is also the relative price of capital with respect to consumption goods.[6] A simple interpretation of parameter q is that it could represent the level of complexity of a general-purpose technology, following the line of thought presented by David (1975 and 1990). This new technology would only be internalized with the acquisition of new machinery and equipment, as, for example, in the dynamo era or the recent ICT revolution with the acquisition of the most recent computers. The adoption of new technologies from the ICT revolution would entail the purchase of new machinery and equipment; on the other hand, there would also be a need to invest in training and redesigning the production lines. In this article, the simplifying hypothesis is that the adoption costs of this technology are proportional to parameter q. Thus, parameter q has a key role in the present model, since it

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[5] The AK model has been used in the discussion on spatial dynamics and other dimensions of growth, such as the time to build. This is mainly because it has flexibility and is able to generate an explicit derivation of welfare analysis. (Boucekkine et. al., 2019; Brito, 2022; Bambi, 2008). For a critical evaluation of the AK model, see Hussein and Thirlwall (2000).

[6] The two-sector model with embodied technology implies that the price of capital goods is equal to $1/q$ in terms of the final good. This relationship illustrates how it is possible to explain the relative price of machines and equipment reduction. It should be noted that the law of motion of capital is expressed in terms of the efficiency units of capital. For more details see Gordon (1990), Greenwood and Krusell (2007), Boucekkine, de la Croix, Licandro, (2011)

stands for the complexity level of the embodied technology, but also represents the amount to be learned (*q*), the gap between the technological edge and the current knowledge in the economy of the technology. The assumption enables us to account for the fact that the more complex the technology, the higher the technological gap, and thus, higher adoption costs. A second process is very important in the dynamics of this model. As time passes, the economy learns how to master the technology and diffuses best practices throughout the economy. As diffusion and learning occur, the technological gap is reduced at a rate of λ. Production, *Y(t)*, is defined as follows, incorporating these features:

$$Y(t) = A(t)K(t) \tag{1}$$

where,

$$A(t) = 1 - qe^{-\lambda t} \tag{2}$$

Therefore[7]:

$$Y(t) = \left(1 - qe^{-\lambda t}\right)K(t) \tag{3}$$

The complete optimization problem for this optimal growth model is given by:

$$V(t) = \max_{C,I} \int_0^\infty e^{-\rho t} \ln C(t)\, dt$$

s.t. (4)

$$Y(t) = \left(1 - qe^{-\lambda}\ \right)K(t)$$

$$\dot{K(t)} = qI(t) - \delta K(t)$$

$$Y(t) = C(t) + I(t)$$

$$K(0) given$$

The objective of the central planner is to maximize the discounted amount of instantaneous utility, where parameter ρ represents the discount factor, *C(t,)* the consumption of the final good. The model includes two sectors, the final and capital goods, respectively, *C(t)* and *K(t).* This specification, similar to that used by Greenwood, Hercowitz, and Krusell (1997), captures the fact that new technologies are embodied in new machines via the law of motion for capital. In other words, technological progress is also realized through the acquisition of new capital goods, and parameter q acts as a measure of the efficiency of the production of capital goods. It is assumed that this stock of capital suffers depreciation at a depreciation rate of δ. To solve the model, the Hamiltonian is defined as:

$$H(t) = e^{-\rho t} \ln\big(C(t)\big) + \mu(t)\big[q\big(1 - qe^{-\lambda t}\big)K(t) - qC(t) - \delta K(t)\big] \tag{5}$$

[7] This specification is not novel in the literature; Greenwood and Jovanovic (2001) employed it to elucidate the diffusion of new technologies. Boucekkine (2000) also utilized this specification in investigating productivity slowdown. Another application of a similar specification in the recent literature is the work by Juhász, R., Lane, N. & Rodrik, D. (2023), where the authors applied this formulation to analyze the efficiency of industrial policy.

where μ is the co-state variable or the shadow price of the capital.

Proposition 1. *By applying the Pontryagin maximum principle to solve (5), it is possible to find the complete dynamics of consumption and capital stock:*

$$C(t) = \frac{\rho}{q} K(0) e^{(q-\delta-\rho)t + \frac{q^2}{\lambda}(e^{-\lambda t}-1)} \quad (6)$$

$$K(t) = K(0) e^{(q-\delta-\rho)t + \frac{q^2}{\lambda}(e^{-\lambda t}-1)} \quad (7)$$

Consumption growth has two components. The first is determined by the long-run growth rate, while the second reflects the effect of the learning curve on the expanding capability of the economy to obtain a better mastering of the adopted technology. It is evident that the faster the diffusion/learning parameter λ, the faster the economy will reach the technological frontier or make best use of the current technology. It should be noted that q appears squared in the equation. The role of q is two-fold: it represents the level of embodied technological progress and also affects the amount that needs to be learned, i.e. the adoption cost, which impacts the entire economy. In other words, the higher the complexity of the technology (q), the higher the adoption cost will be. The economy will have to put in more effort to adopt this more complex technology, due to a higher q. An increase in the adoption cost may be considered a disembodied impact because it affects the marginal productivity of the total installed capital stock. This finding is consistent with Boucekkine et al. (2003), whose model demonstrates that the growth rate of the economy is influenced by the disembodied technological parameter multiplied by the embodied technological parameter weighted by the capital share. In this *AK* model, the share is equal to one. The squared parameter q accounts for both the disembodied (adoption cost) and the embodied impact. It should be observed that q also has an impact on the long-run growth, in the first component, and states that a more complex technology, a higher q, implies a higher long-run growth. The following sub-section analyzes this process in greater detail.

3.1 The Growth Rate

As we have the full dynamics of capital and consumption, it is possible to analyze the components of growth in this economy. Due to the AK structure, the growth rate of *K(t)* and *C(t)* are equal even in the short run. The *AK* model implies that the ratio $\frac{C(t)}{K(t)}$ is constant and equal to $\frac{\rho}{q}$. Over time, as the economy learns about the new technology, the growth rate of consumption and capital become an increasing function of time. The great attribute of this model is that in the transition to the long run, the growth engine contains three components: the complexity level of the technology, capital accumulation and learning and diffusion.

Corollary 1. *From the first-order condition, it may be derived that:*

$$\gamma_K(t) = \gamma_C(t) = (q-\delta-\rho) - q^2 e^{-\lambda} \quad (8)$$

The output growth rate is equal to the sum of the marginal productivity gains through learning, plus the accumulation of capital:

$$\gamma_Y(t) = \gamma_A(t) + \gamma_K(t) = \frac{\lambda q e^{-\lambda t}}{A} + (q-\delta-\rho) - q^2 e^{-\lambda t} \quad (9)$$

The long-run growth rate of consumption and capital is defined as:

$$\lim_{t\to\infty} \gamma_K(t) = \lim_{t\to\infty} \gamma_C(t) = \gamma^* = q - \delta - \rho \qquad (10)$$

In the balanced growth path, in the long run, all variables grow at the same rate, including output:

$$\lim_{t\to\infty} \gamma_Y = \gamma^* = q - \delta - \rho \qquad (11)$$

Equation (8) is important because contrary to the current literature on the misallocation of capital, negative growth may appear as an optimal response to adopting very complex technology. The result allows for the possibility of negative growth, where the level of consumption and output may decrease. This occurs when the technology is sufficiently complex to generate a level of adoption cost, which may not be fully compensated by an increase in the marginal productivity. In short, it may be optimal for the economy to present a transitory drop in consumption and capital accumulation, but this is not a misallocation of capital, it is due to the processes of adoption and learning. Because of the nondecreasing returns to scale, changes in capital stock do not reduce marginal productivity. In this case, a negative growth occurs. As the learning and diffusion processes evolve, the marginal productivity of capital increases and the economy recovers.

This long-run growth (10) is exactly that of a traditional *AK* model. In the long run, the gap is closed and the economy has fully learned how to master the technology. However, the long-run growth will depend on the level of technology that the country has adopted. As $A(t) \to 1$, when $t \to \infty$, the learning and diffusion processes have no impact on the rate of the long-run growth, given that the economy, in the long run, knows how to fully master the adopted technology q. The economy converges to the usual growth rate of the *AK* model.[8] It should be noted, however, that when the technology is more complex, i.e., a higher q, this signifies a higher long-run growth.

4. Negative Growth and welfare implications

The central question therefore, is: under which conditions does the economy present a negative growth, and thus, what are the welfare implications of this negative short-run drop in consumption and capital? Given the optimal rate of growth in this model, it is relatively easy to derive the conditions for observing a negative growth in the economy. Proposition 2 summarizes all the cases. The first step is to determine the level of the negative growth. The second step, given the structural parameters of the economy, is to determine how long this negative growth will last. It should be noted that the diffusion rate takes on a key role in the duration of the negative growth. The level of the technology, q, greatly affects the magnitude of the negative growth.

Proposition 2. *A transitory negative growth rate for consumption and capital will appear for t=0, if:*

1) *Case 1:* $\delta + \rho > 0.25 \Rightarrow \gamma_K(0) = \gamma_C(0) < 0, \forall q$
2) *Case 2:* $0 < \delta + \rho < 0.25 \Rightarrow \gamma_K(0) = \gamma_C(0) < 0 \; if \; q < \frac{1-\sqrt{1-4(\delta+\rho)}}{2}$ *or* $q > \frac{1-\sqrt{1-4(\delta+\rho)}}{2}$

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[8] It should be observed that parameter q should be restricted, so that it is strictly greater than $\delta + \rho$, and lower than 1 (with 1 being the technological edge). The former assures a positive long-run growth and the existence of a balanced growth path, and the latter avoids the possibility of the output level being negative.

3) *Case 3:* $\delta + \rho = 0.25\ \gamma_K(0) = \gamma_C(0) < 0\ if\ q \neq 0.5$

Define $\bar{t}$ *as the time interval that the negative growth lasts, i.e., the duration of the negative growth rate, so the faster the diffusion, the shorter the duration of the negative growth,* $\frac{\partial \bar{t}}{\partial \lambda} < 0$.

By analyzing different scenarios, if the discount rate plus the depreciation rate are too high (above 0.25), the initial cost of adopting new technology becomes a huge burden for the economy, resulting in negative growth rates for consumption and capital accumulation. Conversely, in societies with lower discount and depreciation rates, the complexity level of the technology determines whether the initial negative growth rates are observed. It is simple to analyze the cases. If the discount rate plus the depreciation rate are too high (higher than 0.25), then the economy has a greater preference for the present consumption, so the initial impact of the cost to adopt the technology is a huge burden to the economy, hence the negative rate may be observed.

Output growth dynamics are driven by the sum of productivity gains from the adoption and diffusion of new technology, as well as capital accumulation. In some cases, productivity gains may dominate, leading to positive growth rates above the long-run level. Even when capital growth rates are negative, output growth rates may remain positive and exceed the long-run value. This may be observed by rearranging Equation (9):

$$\gamma_y(t) = \frac{q(\lambda + q^2 e^{-\lambda t} - q)}{e^{-\lambda t} - q} + q - \rho - \delta \qquad (12)$$

The first term on the right hand side converges to zero and the output growth rate is equal to the long-run growth rate. However, the economy may present an output rate above this long-run growth level if the productivity gains offset the adoption cost. Another interesting feature is that the economy may present a non-monotonic convergence toward the *AK* level of long-run growth, depending on the diffusion parameter, *q*. Indeed, the derivative of the output growth rate with respect to time is defined as:

$$\frac{d\gamma_y(t)}{dt} = -\frac{q\lambda^2 e^{-\lambda t}}{(e^{-\lambda t} - q)^2} + q^2 \lambda e^{-\lambda t} \qquad (13)$$

The sign of this derivative might well change. This leads to the possibility of a non-monotonic transition. For example, an economy may present a non-monotonic transition when marginal productivity, *A(t)*, is very low, which leads to a negative growth rate in consumption and capital stock. An *A(t)* decreasing level of total capital stock might imply a decrease in output growth. If the learning and diffusion curve is very steep, the output may have a positive growth rate. Again, as capital accumulation increases so the economy recovers and the output rate rises.

Briefly, the dynamics of output growth depends on the gains of these two components in total output through an improved A and capital stock growth rates.

Proposition 3. *The dynamics of the output growth will be above or below the long-run value in t=0, according to the following three cases:*

Case 1*:* $\lambda > 0.25$. *The diffusion and adoption of the new technologies are very high level. Hence, whatever the value of q, respecting the conditions for a positive long-run growth rate, the dynamics of productivity gains prevail over the capital.*

Case 2: $\lambda < 0.25$*. The adoption and diffusion of new technologies are not high enough to dominate the pattern of dynamics of the capital accumulation. In this case, if the growth of output increases with time, it converges with the long-run level. Conversely, if* q *is outside the interval, the dynamics of diffusion dominate and the output growth rate will be on a long-run level and productivity gains will exactly offset the impact of capital accumulation.*

Case 3: $\lambda = 0.25$ *if* $q = 0.25$*, the economy is on a long-run level, the output growth rate is constant and equal to* $q - \rho - \delta$*. Otherwise, if* $q \neq 0.25$*, the dynamics of the output growth will be driven by the diffusion of the new technologies.*

The question that should be asked is: can there be a negative growth on total output? Alternatively, which q values yield a negative output growth rate?

Corollary 2. *A negative output growth rate may be observed in some cases in which capital accumulation drives the output growth process.*

We may observe a simple case where t=0, and rewrite the output growth rate as in *t=0*:

$$\gamma_y(0) = \frac{q\lambda + (1-q)(q-\rho-\delta-q^2)}{1-q} = \frac{q\lambda + (1-q)\gamma_k(0)}{1-q}$$

then the condition for a negative output growth requires the numerator to be negative. In other words, a drop in the capital stock weighted by the technological gap should be greater, in modulus, than the gains through diffusion:

$$q\lambda + (1-q)\gamma_k(0) < 0$$

When $\gamma_k(0) < 0$, i.e., there is a negative growth in the capital, the output will have a negative growth if:

$$\lambda < -\frac{(1-q)}{q}\gamma_k(0)$$

In short, it is possible to obtain an optimal reduction in output, consumption and capital accumulation, due to adoption cost. Furthermore, it is also possible to derive the time that this negative growth will last. This reduction will unequivocally be reduced since the learning/diffusion is higher. It is possible to demonstrate either that the complexity level, q, will have an ambiguous effect, or that there is a threshold level above which the optimal time of the negative growth will be reduced. The model may also present a non-monotonic behavior on the output growth, capital and consumption. It is therefore crucial to discover what the welfare implications are for all this cases.

4.1 Welfare function and some comparative statistics

The different cases that the model may present spark a discussion regarding which optimal choices policy makers must make. Given that there is an explicit solution for consumption, it is also possible to obtain an explicit welfare function.

Proposition 4. *By taking the path of consumption from the first-order conditions, the discounted amount of the instantaneous utility is equal to:*

$$V(t) = \int_0^{\infty} e^{-\rho t} \ln C(t)dt = \int_0^{\infty} e^{-\rho t}\left[\ln \frac{\rho}{q} K(0) - \frac{q^2}{\lambda} + (q-\rho-\delta) + \frac{q^2}{\lambda} e^{-\lambda t}\right] dt \quad (14)$$

which yields:

$$V(t) = \frac{\ln\left(\frac{\rho}{q}K(0)\right)}{\rho} - \frac{q^2}{\lambda\rho} + \frac{q-\rho-\delta}{\rho^2} + \frac{q^2}{\lambda(\lambda+\rho)} = \frac{\ln\left(\frac{\rho}{q}K(0)\right)}{\rho} + \frac{q-\rho-\delta}{\rho^2} - \frac{q^2\lambda}{\lambda\rho(\lambda+\rho)} \quad (15)$$

Interpreting this welfare function is a very simple and direct task. It is the sum of the present value of the long-run steady state given by the two first terms on the right-hand side of the welfare equation (15), plus the discounted impact of growth on total welfare. Due to the transition toward the long-run, the total impact of the growth rate may be decomposed into the long-run discounted rate, given by $\frac{q-\rho-\delta}{\rho^2}$, plus a convergence toward the long-run, given by $\frac{q^2}{\lambda(\lambda+\rho)}$. From the welfare function, it is possible to derive some comparative statics, whereby the main aim is to define the impacts of learning and diffusion, λ, and the complexity level, q.

Corollary 3. *The diffusion process has an unambiguous positive effect on welfare, however the impact of the complexity level will be ambiguous:*

$$\frac{\partial V(t)}{\partial \lambda} = \frac{q^2}{\rho(\lambda+\rho)^2} > 0$$

$$\frac{\partial V(t)}{\partial q} = -\frac{1}{\rho q} + \frac{1}{\rho^2} - \frac{2q}{\rho(\lambda+\rho)}$$

The impact of λ on welfare is unambiguously positive. The faster the diffusion or the learning parameter, the higher the welfare gains. Parameter q has a much richer impact on *V(t).* On the one hand, it increases the long-run growth, although on the other, a greater amount to be learnt signifies a greater distance to the frontier. The sign of the derivative is ambiguous and is dependent on the value of other parameters.

To study the impact of a new technology on the economy, q, some comparative statics may be used to study the marginal impact of implementing an improved technology.

In the long run, an improvement in the technology brings an unambiguous positive impact on the growth rate:

$$\frac{\partial \gamma^*}{\partial q} = 1 > 0$$

However, in the short run, the introduction of a more complex technology may reduce growth, since the adoption cost will play a role in reducing the initial output:

$$\frac{\partial \gamma_C(t)}{\partial q} = \frac{\partial \gamma_k(t)}{\partial q} = 1 - 2qe^{-\lambda t}$$

If $q < 0.5$, then the derivative will always be positive and the marginal impact of an improved technology is positive for $t>0$. However, for more complex technologies $q \geq 0.5$, the marginal impact on growth may be negative in the time interval $0 < t < -\frac{1}{\lambda}\ln\frac{1}{2q}$.

The negative impact of adopting a more complex technology over the short run can lead to a reduction in welfare. This is because despite the long-term increase in growth, society incurs an initial cost to implement the new technology. Therefore, a very complex technology may not necessarily improve welfare, since it depends on the parameters of the utility function and how fast the economy learns.

In order to illustrate the flexibility of the model, two simulations are presented. The first simulation compares how different levels of initial capital and income affects the trajectory of some endogenous variable, notably welfare. As it is possible to have the full dynamics of capital, consumption, and welfare in the model, the simulations are presented along with the trajectory of the level of the variables. Figure 1 illustrates this impact on welfare when a higher level of technology is introduced into the economy. In this first simulation, there are two economies, and the structural parameters remain the same, as follows: $\rho=0.1$; $\delta=0.1$; $\lambda=0.03$. The only distinction between the two economies lies in their initial capital, whereby $K(0)$ is set at 550 for the baseline economy and a lower initial capital of 450 for Series 2. The numbers are chosen just to demonstrate the flexibility of the model and to illustrate the implicit dynamics. The second simulation aims to study the impact of learning on welfare, comparing two countries with the same level of initial capital.

The first simulation demonstrates that the shape of the welfare is similar for both countries, when the level of level of technological complexity varies. However, the country with a lower initial capital presents a lower welfare at every level of technological complexity.[9] The figure clearly illustrates a U-shaped relationship between the level of complexity (q) and welfare, indicating an optimal q level. While a more complex technology enhances long-term growth and welfare, excessive complexity results in a decline in welfare due to increased learning and adoption costs

In short, countries with different levels of initial capital will present a very similar dynamic behavior, although the level of welfare will differ with a relatively wealthier country presenting a higher level of welfare. There is a non-monotonic relationship between welfare and the complexity level of the technology, thus requiring economies to balance short-term costs and long-term gains.

[9] The condition for the existence of a balanced growth path is that q is higher than the sum of the discount rate and the depreciation rate.

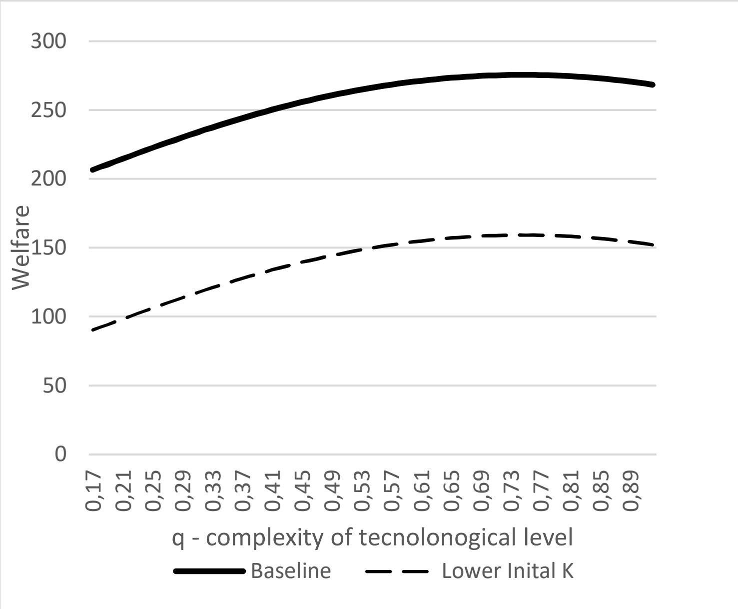

Figure 1 – Level of complexity of the technology and welfare for two economies with different initial capital.

Baseline: K(0) =550; $\rho = 0.1$;$\delta = 0.1$;$\lambda = 0.03$;

Lower Initial K: K´(0) =300; $\rho = 0.1$;$\delta = 0.1$;$\lambda = 0.03$

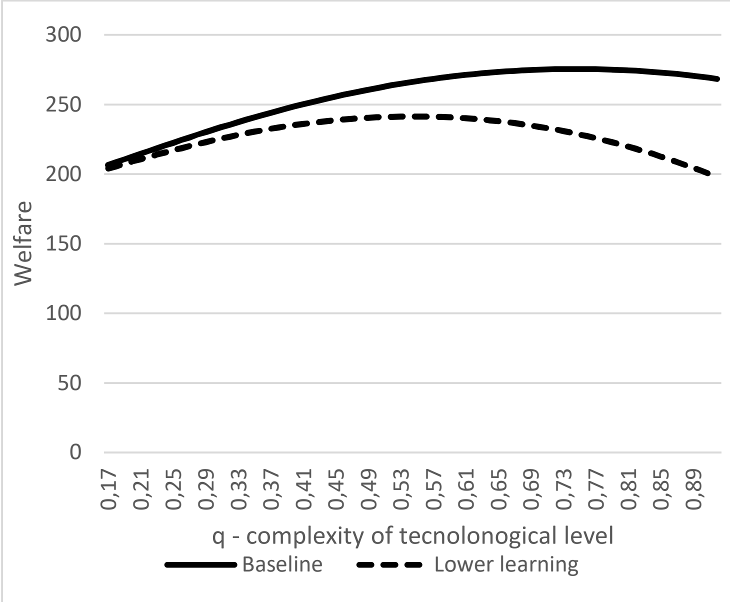

Figure 2 – Complexity level of the technology and welfare for two economies with different learning and diffusion parameters

Baseline: K(0) =550; $\rho = 0.1$;$\delta = 0.1$;$\lambda = 0.03$;

Country 3: K´(0) =550; $\rho = 0.1$;$\delta = 0.1$;$\lambda = 0.01$

The second simulation presents the impact of learning and it can be seen in figure 2. There is a baseline economy, as in the previous simulation, and an economy with a lower learning and diffusion process. This 'lower learning economy' has the same initial capital as the baseline economy, of 550, however, there is a lower learning parameter, $\lambda=0.01$. In the second simulation, the baseline economy is the same, but there is another economy with the same level of initial capital, 550, although $\lambda=0.01$. This economy is called the lower learning economy. The U-shaped relationship between the level of complexity (*q)* and welfare is evident from the figure 2, indicating an optimal level of q. While a more complex technology increases long-run growth and improves welfare, an excessively complex technology leads to a welfare loss due to the learning and adoption costs. Figure 2 presents the results of the welfare level, as the complexity of the technology varies.

There is still a U-shaped relationship between welfare and the complexity level of the technology. However, the economy with a lower level of learning and diffusion also presents a lower optimal complexity level of technology. At the same level of complexity of a given technology, the baseline economy enjoys a higher level of welfare. Furthermore, as the level of technological complexity increases, the welfare gap between the baseline economy and the lower learning level widens. In other words, the economy with a higher level of learning and diffusion is able to enjoy higher levels of welfare with more complex technologies since it not only grows faster but is also able to offset the negative impact of adopting complex technologies more quickly. Learning is a key variable in the model, since it is welfare improving and allows the economy to adopt more complex technologies and provide higher levels of growth and output.

5. *Leapfrogging and the transition dynamics*

Although the model has a simple structure, it is capable of generating rich dynamics. To illustrate this, we undertook another simple illustration considering two economies: Economy 1 adopts a more complex technology but has a lower learning parameter (*q=0.265*) and a slower learning rate *(λ=0.15*), while Economy 2 adopts a less complex technology but learns much faster (*q=0.26, λ=0.28*). Both economies share the same structural parameters, *δ=0.15* and *ρ=0.1*, but present different long-run growth rates of 1.5% and 1%, respectively. Figure 3 displays the dynamics of the output growth rate for both economies. Initially, Economy 2 starts at a higher growth rate due to its faster learning, while Economy 1 experiences a temporary reduction in total output. However, by period 12, Economy 1 has caught up and begins to grow faster than Economy 2, resulting in a leapfrogging effect. It is interesting to note that the convergence dynamics are entirely different for each country.

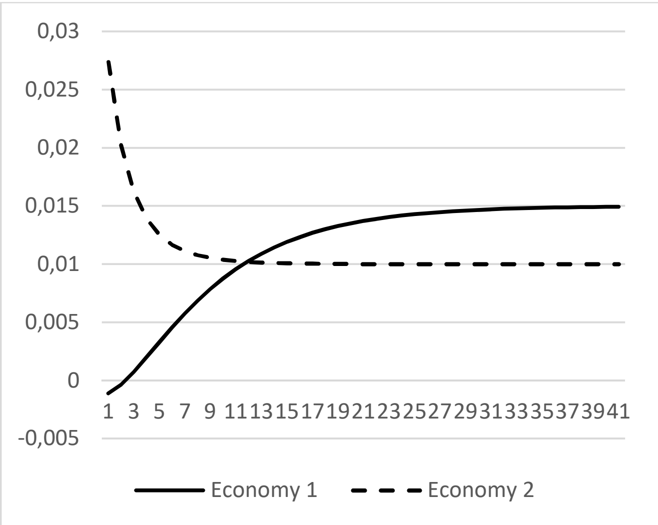

Figure 3 – Output and growth rate for two economies: Economy 1 adopting a more complex technology, but with lower learning; Economy 2 adopting a less complex technology, but with faster learning and diffusion

Source: Simulated data

Parameters:

Economy 1: $q_1 = 0.26; \lambda = 0.15$

Economy 2: $q_2 = 0.265; \lambda = 0.28$

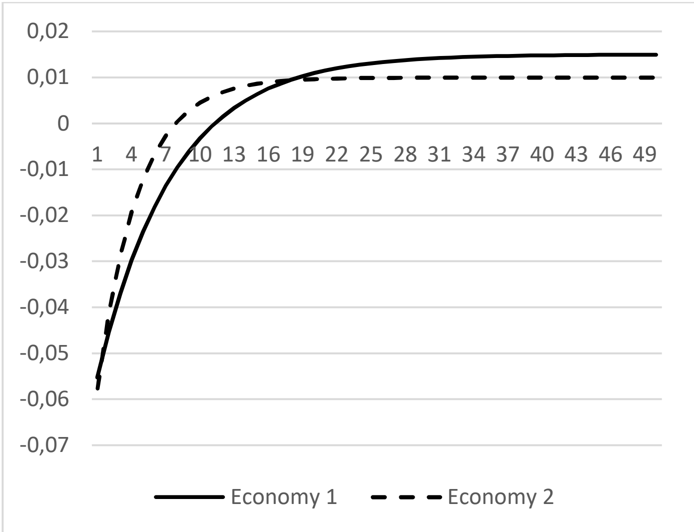

Figure 4 - Consumption and capital growth rate for Economy 1 and Economy2

Source: Simulated data

Parameters:

Economy 1: $q_1 = 0.26; \lambda = 0.15$

Economy 2: $q_2 = 0.265; \lambda = 0.28$

In both economies, there is a decline in consumption and capital stock accumulation because the adoption costs exceed the marginal productivity of capital. In the initial periods, both economies experience negative growth rates for consumption and capital stock accumulation. However, in Economy 2, where the learning and diffusion processes are greater, the economy recovers faster than in Economy 1. Figure 4 illustrates this dynamic. The decrease in consumption and capital stock accumulation is much less severe in Economy 2 than in Economy 1, which experiences a more persistent reduction due to the higher adoption cost of more complex technology. A positive growth is only presented after 10 periods. Nevertheless, due to its higher long-run growth rate, Economy 1 leapfrogs Economy 2's growth rate of consumption and capital stock accumulation of around $t=17$.

The key message is that all these variables are interrelated, and a mix of policies may provide higher growth and improved welfare. By understanding the trade-offs and synergies among these factors, policymakers may design more effective policies that promote sustainable economic development.

6. Conclusions

The recent challenges to developing economies have been the so-called middle income trap and a reduction in the diffusion of new technologies, as well as a slowdown in productivity growth. Gills and Kharas (2015) argued that traditional new neoclassical models, such as Solow, are unable to address this reality. At the same time, endogenous growth models state more about the production of new ideas or technologies than their adoption. In addition, the economic literature emphasizes the crucial role of learning and the diffusion of new ideas and technologies in driving growth, particularly in developing countries. This paper has presented a simple model that incorporates some of these features. Using an AK model with embodied capital technology (i.e., new ideas or technologies are embodied in capital goods), the model incorporates an important characteristic of technology adoption: it is costly to introduce a new technique or machine in production. To encapsulate this concept, the model incorporates a Nelson-Phelps catch-up equation, yielding intriguing findings:

(a) The possibility of catching up and leapfrogging within an AK model framework;

(b) The potential for a negative growth and a non-monotonic transition toward a balanced growth path due to adoption costs;

(c) A shorter duration of the negative growth with higher rates of learning and diffusion, although the impact of technological complexity remains ambiguous;

(d) A trade-off exists for economies, whereby adopting a more complex technology enhances long-term growth but results in a short-term reduction in productivity.

The simulation and comparative statistics presented in the present paper have illustrated that the optimal pace of adopting technology creates a trade-off between short-run costs and long-run benefits. These results are particularly relevant for policy makers in developing countries, since some of the benefits of adopting technology may only materialize once the economy learns more about the new technology. Countries may face the choice of whether to adopt a more complex technology with higher short-run costs but with greater long-run gains. These results are significant and provide a strong contribution to the discussion on broader industrial policies. Over recent years, the focus of the economic literature has concentrated more on how to implement these policies rather than whether to create them. (Juhász, Lane and Rodrik, 2023) The paper has also illustrated how a mix of policies may prove to be welfare enhancing. There is a variety of instruments that may be analyzed with this model, ranging from more horizontal interventions, such as reducing the adoption cost, as undertaken by Darpa with the Mead-Conway revolution and VLSI, to a more traditional intervention, such as capital accumulation subsidies or improving learning and diffusion. One important result of the model is that when adopting a more complex technology, in order to be welfare enhancing it is desirable to combine it with diffusion and learning improvement.

The model generates rich dynamics and various transitional paths toward a long-run equilibrium rate. The simulations have demonstrated that even an economy, which adopts a lower level of technology may outperform another economy with a more complex technology but which is less efficient in learning and diffusing new ideas. The richness of the model suggests a range of possible interventions that could improve welfare. One natural extension of this study would be to incorporate optimal intervention and the path of government expenditures.

One other significant result of the model is the possibility of an optimal negative growth. This reduction in productivity growth is an optimal response in the economy due to adoption cost and the level of complexity of the adopted technology. However, the dynamics of technological diffusion and learning reduce this initial loss, and translate it to a positive long-run growth. This results challenges the traditional interpretation of capital misallocation[10]. The static inefficiency and productivity may be the result of an optimal choice and the transition to a sustainable growth process.

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Appendix

Proof of Proposition 2. First, it is straightforward to show that γ_C is a monotonic increasing function of time, $\frac{\partial \gamma}{\partial t} = \lambda q^2 e^{-\lambda t} > 0$.

To establish the value of q, where $\gamma_C < 0$, it is sufficient to study the case where t=0, since γ_C is a monotonic increasing function of time. Thus, it is necessary to solve the polynomial in q:

$$q - (\delta + \rho) - q^2 = 0$$

The roots will be given by:

$$q = \frac{1 \pm \sqrt{1 - 4(\delta + \rho)}}{2}$$

Therefore, to obtain the level of the technology that generates a negative growth, it is necessary to study 3 different cases: first, no real roots, second, two real roots and third, one unique real root:

Case 1: if $\delta + \rho > 0.25$, there are no real roots, and since parameter q^2 has a negative sign, irrespective of the value of q, the growth rate will be negative.

Case 2: if $\delta + \rho < 0.25$, there are two real roots for which the initial growth rate will be equal to 0. As q^2appears with a negative sign, the region that $\gamma_C(0) < 0$ is in the interval $\left]0, \frac{1-\sqrt{1-4(\delta+\rho)}}{2}\right[$ and $\left]\frac{1+\sqrt{1-4(\delta+\rho)}}{2}, 1\right[$

Case 3: $\delta + \rho = 0.25$, there is just one real root, q=0.5, and outside this point the value of γ_C will be negative.

Finally, if there is a negative growth $\gamma_C < 0$, then in order to obtain $\bar{t}$, that is how long the negative growth will last, it is sufficient to find the value $\gamma_C(t^* = \bar{t}) = 0$, since the growth rate is increasing in time. Substituting this in (8) and making $\gamma_C(t^* = \bar{t}) = 0$, yields:

$$\bar{t} = -\frac{1}{\lambda} \ln\left(\frac{q - \rho - \delta}{q^2}\right)$$

Differentiating with respect to λ:

$$\frac{\partial \bar{t}}{\partial \lambda} = \frac{1}{\lambda^2} \ln\left(\frac{q - \rho - \delta}{q^2}\right) < 0$$

Because $\left(\frac{q-\rho-\delta}{q^2}\right) < 1$ when t>0.■